SCIENTIFIC MEMOIRS

EDITED BY

J. S. AMES, Ph.D.

PROFESSOR OF PHYSICS IN JOHNS HOPKINS UNIVERSITY

X.

THE WAVE-THEORY OF LIGHT

Schuyler B Serviss
April 11, 1906

THE

WAVE THEORY OF LIGHT

MEMOIRS BY HUYGENS, YOUNG AND FRESNEL

EDITED BY

HENRY CREW, PH.D.
PROFESSOR OF PHYSICS, NORTHWESTERN UNIVERSITY

NEW YORK .:. CINCINNATI .:. CHICAGO
AMERICAN BOOK COMPANY

Crew, Light.
W. P. I

PREFACE

THANKS to the labors of Kirchhoff, Kelvin, Huxley, and others, there is now a widespread opinion that any physical phenomenon is "explained" only when some one has devised a dynamical model which will duplicate the phenomenon. The completeness of the explanation is to be measured by the completeness with which the model will duplicate the phenomenon. Thus, for instance, a refraction model which, like that of Airy, describes only the path of the refracted ray when the incident ray is given, does not in any true sense explain how the refracted ray comes to take one path rather than another. Such a model illustrates Snell's law, but does not explain the phenomenon.

If, however, we take a large and shallow tank of water, the floor of the tank being partly covered with a false bottom, so as to give two, and only two, different depths of water, we shall find that the speed of the waves in the deeper portion of the tank bears to the speed in the shallower portion a constant ratio; hence, in passing from one depth to the other, these waves are refracted according to the sine law.

Such a model may be said to be a "partial explanation" of refraction in so far as it refers the phenomenon to change of speed which accompanies change of medium. It represents, however, only the kinematics of refraction.

If, now, we could go one step further, and make a model in which the wave-producing forces were duplicated—in other words, if we could make a model of the medium and of the disturbing forces—we should have a fairly complete "explanation" of refraction; in fact, the dynamics of refraction would be understood. This would imply not only that we knew the substance disturbed, but also that we were acquainted with the laws according to which it is disturbed.

PREFACE

A theory of light may be considered either from a kinematical or from a dynamical point of view. To assume, on experimental grounds, that a ray of light has a certain speed in one medium and a different speed in a different medium, and that it consists in a particular kind of motion, and *thence* to infer the laws of refraction, rectilinear propagation, and diffraction, is to construct a kinematical theory of light. But to assume a certain structure for the luminous body and for the medium, and thence to derive the motions and the different speeds assumed in the kinematical case, is to offer a dynamical explanation of light.

The wave-theory of light is used, nearly always, in the former and narrower sense to mean the kinematical explanation of light; it leaves entirely to one side the dynamical questions hinted at above. It assumes, not without strong experimental evidence, the existence of waves travelling with different speeds in different media, and proposes to explain the cardinal phenomena of optics.

To illustrate its limitations, we may cite the instance of the ordinary and extraordinary ray in crystals. How it happens that there are two rays is a problem in the dynamics of light; but, assuming these two rays, their subsequent behavior, their inability to interfere, etc., must be accounted for—in a general way, at least—by the kinematical theory of light.

It is in this narrow sense that the wave-theory of light is employed in the memoirs translated in this volume.

The first clear and unmistakable suggestion that light consists in a vibratory motion appears to be due to that brilliant but unfortunate genius, Robert Hooke (1635–1703), who, in his *Micrographia* (London, 1665), describes the three characteristic features of the motion which he believes to constitute light.

Since it has not been deemed advisable to reprint Hooke's paper in this volume, it may not be out of place here to quote what few paragraphs are necessary fairly to present his point of view. This will, perhaps, be accomplished by the following selections:

"It would be somewhat too long a work for this place *Zetetically* to examine, and positively to prove, what particular kind of motion it is that must be the efficient of Light; for though it be a motion, yet 'tis not every motion that produces it, since we find there are many bodies very violently mov'd, which yet afford not such an effect; and there are other bodies, which to our senses, seem not mov'd so much, which yet shine. Thus Water and quick-silver, and most other liquors heated, shine not; and several hard bodies, as Iron, Silver, Brass, Copper, Wood, &c., though very often struck with a hammer, shine not presently, though they will all of them grow exceeding hot; whereas rotten Wood, rotten Fish, Sea Water, Gloworms, &c. have nothing of tangible heat in them, and yet (where there is no stronger light to affect the Sensory) they shine some of them so Vividly, that one may make a shift to read by them.

"It would be too long, I say, here to insert the discursive progress by which I inquir'd after the proprieties of the motion of Light, and therefore I shall only add the result.

"And, First, I found it ought to be exceeding *quick*, such as those motions of *fermentation* and *putrefaction*, whereby, certainly, the parts are exceeding nimbly and violently mov'd; and that, because we find those motions are able more minutely to shatter and divide the body, then the most violent heats or *menstruums* we yet know. And that fire is nothing else but such a *dissolution* of the Burning body, made by the most *universal menstruum* of all *sulphureous bodies*, namely, the Air, we shall in an other place of this Tractate endeavour to make probable. And that, in all extremely hot shining bodies, there is a very quick motion that causes Light, as well as a more robust that causes Heat, may be argued from the celerity wherewith the bodyes are dissolv'd.

"Next, it must be a *Vibrative motion*. And for this the newly mention'd *Diamond* affords us a good argument; since if the motion of the parts did not return, the Diamond must after many rubbings decay and be wasted; but we have no reason to suspect the latter, especially if we consider the exceeding difficulty that is found in cutting or wearing away a Diamond. And a Circular motion of the parts is much more improbable, since, if that were granted, and they be suppos'd irregular and Angular parts, I see not how the parts of the Diamond should hold so firmly together, or remain in the same sensible dimensions, which yet they do. Next, if they be *Globular*, and mov'd only with a *turbinated* motion, I know not any cause that can impress that motion upon the *pellucid medium*, which yet is done. Thirdly, any other *irregular* motion of the parts one amongst another, must necessarily make the body of a fluid consistence, from which it is far enough. It must therefore be a *Vibrating* motion.

"And Thirdly, That is a very *short vibrating motion*, I think the instances drawn from the shining of Diamonds will also make probable. For a Diamond being the hardest body we yet know in the World, and consequently the least apt to yield or bend, must consequently also have its vibrations exceeding short.

"And these, I think, are the three principal proprieties of a motion, requisite to produce the effect call'd Light in the Object."—[*Micrographia*, pp. 54–56.]

The total absence of experimental evidence from the above statement of the case stands in such marked contrast with the method of modern physics as initiated by Galileo, that we cannot for a moment reckon Hooke among the founders of the wave-theory.

So important, on the contrary, have been the contributions of Huygens, Newton, Young, and Fresnel, that each has in turn been considered the founder of the modern science of optics. What justification there is for each of these views will be clearer from a brief consideration of optical theory before and after it had been modified by the work of each of these four men.

Two questions naturally arise in the consideration of any theory, *viz.*, (1) What phenomena does it explain?—and (2) How does it explain them? The answers which have been given to these two questions at various periods in the development of the wave-theory may be outlined as follows:

At the time when Huygens and Newton began their work on light, the following phenomena were demanding explanation:

1. The existence of rays and shadows, known from the earliest times.
2. The phenomenon of reflection, known from the earliest times.
3. The phenomenon of refraction, as described by Snell's law.
4. The rainbow and the production of color by the prism.
5. The colors of thin plates—Newton's rings.
6. Diffraction bands outside the geometrical shadow, described by Grimaldi, 1665.

To these might be added the two following phenomena which were discovered before the final publication of Newton's *Opticks* (1704) or Huygens's *Traité de la Lumière* (1690).

7. The polarization of light by crystals (Bartholinus, 1670).
8. The finite speed of light (Römer, 1675).

Of these eight cardinal facts, the ***second***, the ***third***, and the ***eighth***, were explained by Huygens on the assumption—

(*a*) That a luminous disturbance consists of a wave-motion in the ether.

(*b*) That this wave-disturbance travels with a uniform finite speed through the ether in any homogeneous medium.

(*c*) That in different media it travels with speeds which are related inversely as the refractive indices of those media.

But the wave-disturbance as pictured by Huygens was a single longitudinal pulse, or blow, imparted to an elastic fluid. Since he did not have in mind either a train of waves or transverse waves, or the idea of "phase," or waves of different lengths, it is evident that he was unable to explain any of the remaining five facts.

Turning now to that portion of the work of Newton which contributed to the wave-theory, we find that the *fourth* phenomenon—prismatic colors—was explained by him in 1666, when he demonstrated that a single ray of white light contains all the colors of the spectrum, and that color is not produced at the surface of the prism, as had been hitherto supposed. This discovery made possible, for the first time, the correct explanation of the rainbow.

In Newton's ingenious, though, as we now know, incorrect explanation of the *fifth* phenomenon—colors of thin plates—we meet the earliest measurement of the wave-length of light, *viz.*, the distance traversed by a ray of light during the interval between two successive "fits" of the same kind. We meet here, also, the first evidence that, in these fits, or, as we now say, waves, there is a regular periodicity. From this point on we must consider light as travelling not only in waves, but in trains of waves.

At the close of the period of Huygens and Newton, we have then the following facts still demanding explanation:

1. The existence of rays and shadows.
5. The colors of thin plates.
6. The existence of diffraction fringes.

7. The polarization of light by crystals.

To these must now be added—

9. The phenomenon of stellar aberration, discovered by Bradley in 1727.

Considering next the work of Young, we find that he first suggested the correct explanation for the colors of thin plates, having shown by experiment that two rays of light can interfere to produce alternately bright and dark bands. From this experiment and the dark centre in Newton's rings, he concludes that light consists of series of waves which, like other wave-motions, change phase by 180° on reflection from a denser medium.

Young, at this period (1802–3), was still laboring under the impression that light-waves were longitudinal and were propagated in a fluid medium; fortunately, neither of these assumptions affects the validity of his reasoning concerning the colors of thin plates.

When Fresnel began his optical studies (1814) the following facts, *viz.*, (1) existence of rays, (6) diffraction fringes, (7) polarization, and (9) aberration, were still to be accounted for on the wave-theory. By the union of Huygens's principle with the principle of interference, Fresnel gave the first satisfactory explanation of the rectilinear propagation of light, and of the existence of diffraction fringes outside the geometrical shadow.

Fresnel's memoir, in which these discoveries are most systematically set forth, and which was "crowned" by the French Academy in 1819, is translated in the following pages. For the purpose of offering an elementary geometrical explanation of rays and diffraction bands, Fresnel invented the idea of dividing the wave-front into a certain series of zones, which in nearly all text-books are wrongly referred to as "Huygens's Zones." That this is not only unfair, but also misleading, has been pointed out by Professor Schuster.—*Phil. Mag.* vol. xxxi., p. 77 (1891). The first mention of these Fresnel Zones, as they should be called, will be found on p. 111 of the present volume.

It was in order to explain the phenomenon of polarization that Fresnel introduced the idea of transverse vibrations in the ether. The boldness of this now universally accepted hypothesis, which was then practically equivalent to supposing the ether an elastic solid, can be fully appreciated only after one has carefully studied the views of Fresnel's contemporaries.

The evidence for the transversality of light vibrations rests upon the inability of two oppositely polarized rays to interfere. The memoir of Arago and Fresnel upon this subject is translated in the present volume.

Of the nine phenomena which we have more or less arbitrarily selected as the principal facts of optics, all, save only the last—aberration—had received a fairly complete explanation at the close of the labors of Young and Fresnel. This discovery of Bradley's, which he so easily disposed of on the corpuscular theory, has received many explanations in terms of the wave-theory; but none of these can be considered as thoroughly satisfactory. Young imagines the ether to pass through ordinary matter "as freely, perhaps, as the wind passes through a grove of trees." On this view, however, it is difficult to see how the speed of light in glass, say, should differ from its speed in a vacuum, or how the aberration constant can remain unchanged when the tube of the telescope is filled with water, as in Airy's experiment.—*Proc. Roy. Soc.*, vol. xx., p. 35 (1872).

For it will be remembered that the aberration constant is v/V radians, where

v=speed of earth in its orbit,

and V=speed of light between the objective and eye-piece of telescope employed.

Fresnel, accordingly, modified Young's hypothesis by assuming that, in their motion through space, refracting bodies carry with them only so much ether as is required to increase the density of free ether from unity to ρ, where ρ at any point in the medium is defined by the following equation:

$$\frac{\rho-1}{\rho}=\frac{\mu^2-1}{\mu^2},$$

μ being the refractive index at the same point in the body.

This is really equivalent to saying that "the luminiferous ether is entirely unaffected by the motion of the matter which it permeates." [*Amer. Jour. Sci.*, vol. cxxxi., p. 386.] And that this is the fact of nature is exactly the conclusion at which Fizeau and Michelson and Morley arrive from their experiments upon the effect of motion of the medium upon the speed of light.—*Loc cit.*, p. 377.

When, however, Michelson and Morley attempt to detect this relative motion of the earth and the ether as the earth proceeds in its orbital motion, they do not succeed in certainly finding that there is any [*Phil. Mag.*, December, 1887]; and they accordingly conclude that this relative motion is "quite small enough to refute Fresnel's explanation of aberration."

Of the two experimental facts just cited, one apparently confirms Fresnel's view, and makes possible an explanation of aberration in terms of the wave-theory; while the other leads us to think that the ether moves with the refracting medium, in which case the wave-theory appears incompetent to explain stellar aberration.

It was in the year 1850 that Fizeau and Foucault measured directly the speed of light in air and in water, and found the ratio of these speeds numerically equal to the ratio of their refractive indices. This experiment has sometimes been called the *experimentum crucis* of the wave-theory; but with scant justice we venture to think, inasmuch as no great doctrine in physics can be said to rest upon any single fact, though modification may be demanded by a single fact.

We have now followed, in merest outline, the general explanations which Huygens, Newton, Young, and Fresnel have offered for all, save one, of this group of nine cardinal facts. It is needless to remind the reader that this enumeration forms but a small fraction of the phenomena which optical science has brought to light within the last two centuries, or, indeed, since the labors of these four men were ended.

No outline of the wave-theory would be complete without mention of the important addition which was made to it in the year 1849 by Sir George Stokes. For he it was who first completely justified Huygens's principle by showing that if the primary wave be resolved as proposed by Huygens, no "back wave" will be produced provided we adopt the proper law of disturbance for the secondary wave. The discovery of this law was announced in his memoir on the *Dynamical Theory of Diffraction.* [*Trans. Camb. Phil. Soc.*, vol. ix., p. 1; *Math. and Phys. Papers,* vol. ii., p. 243.] Mathematically speaking, this contribution amounts to the introduction of the factor $1 + \cos\theta$ into the equation [*Eq.* 46, *loc. cit.*], which describes the disturbance in a secondary wave proceeding from an element of the primary wave.

PREFACE

While, as has been said above, the following memoirs concern themselves only with the kinematics of light-waves and not at all with the question of what is vibrating, it may not be out of place to indicate that principally during the last half of the present century at least four more cardinal facts have presented themselves and demanded explanation.

10. The speed of light in free space is numerically equal to the ratio of the electrostatic and electromagnetic units of quantity.

11. In refracting media, the speed of light varies inversely as the square root of the product of the electric and magnetic inductivities.

12. "Most transparent solid bodies are good insulators, and all good conductors are very opaque."—MAXWELL, *Treatise*, vol. ii., art. 799.

13. The plane of polarization is rotated in a magnetic field. (Faraday.)

It was to "explain" these additional phenomena that Maxwell proposed, in 1865, to modify the wave-theory of light by replacing the mechanical shear of the ether by an electric displacement. How thoroughly justified Maxwell was in this move has been amply proved mathematically by the analogy of his equations with those of the elastic solid theory, and experimentally by Hertz (1888).

Within the last decade the wave-theory has shown itself capable of explaining an entirely new group of phenomena, *viz.*, the color photography discovered by Lippmann. Wiener has shown that we have here merely two rays of light—the direct and reflected travelling in opposite directions and interfering to produce stationary light waves.

The flexibility of the wave-theory has still more recently been exemplified by the beautiful discovery of Zeeman; and Larmor and Preston have shown that by assuming a particular kind of electrical displacement, *viz.*, an orbital motion of an ion, the wave-theory is competent to predict not only the triplets and even the sextet, but also the polarization produced by placing the source of radiation in a magnetic field.—*Phil Mag.*, February, 1899.

Striking as the resemblance appears between the kinematics of wave-motion considered in this volume and the phenomena of optics, it must never be forgotten that in all probability the vibrating atom is a structure whose motion is vastly complicated as compared with the few simple motions which the experiments of Huygens, Newton, Young, Fresnel, Maxwell, and Michelson have assigned to it.

H. C.

EVANSTON, Ill., *November*, 1899.

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BY

CHRISTIAAN HUYGENS

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TREATISE ON LIGHT

BY

CHRISTIAAN HUYGENS

PREFACE

THIS treatise was written during my stay in Paris twelve years ago, and in the year 1678 was presented to the Royal Academy of Sciences, to which the king had been pleased to call me. Several of this body who are still living, especially those who have devoted themselves to the study of mathematics, will remember having been at the meeting at which I presented the paper; of these I recall only those distinguished gentlemen Messrs. Cassini, Römer, and De la Hire. Although since then I have corrected and changed several passages, the copies which I had made at that time will show that I have added nothing except some conjectures concerning the structure of Iceland spar and an additional remark concerning refraction in rock-crystal. I mention these details to show how long I have been thinking about these matters which I am only just now publishing, and not at all to detract from the merit of those who, without having seen what I have written, may have investigated similar subjects: as, indeed, happened in the case of two distinguished mathematicians, Newton and Leibnitz, regarding the question of the proper figure for a converging lens, one surface being given.

It may be asked why I have so long delayed the publication of this work. The reason is that I wrote it rather carelessly in French, expecting to translate it into Latin, and, in the meantime, to give the subject still further attention. Later I thought of publishing this volume together with another on dioptrics in which I discuss the theory of the telescope and the phenomena associated with it. But soon the subject was no longer new and was therefore less interesting. Accordingly I kept putting off the work from time to time, and now I do not know when I shall be able to finish it, for my time is largely occupied either by business or by some new investigation.

In view of these facts I have thought wise to publish this manuscript in its present state rather than to wait longer and run the risk of its being lost.

One finds in this subject a kind of demonstration which does not carry with it so high a degree of certainty as that employed in geometry; and which differs distinctly from the method employed by geometers in that they prove their propositions by well-established and incontrovertible principles, while here principles are tested by the inferences which are derivable from them. The nature of the subject permits of no other treatment. It is possible, however, in this way to establish a probability which is little short of certainty. This is the case when the consequences of the assumed principles are in perfect accord with the observed phenomena, and especially when these verifications are numerous; but above all when one employs the hypothesis to predict new phenomena and finds his expectations realized.

If in the following treatise all these evidences of probability are present, as, it seems to me, they are, the correctness of my conclusions will be confirmed; and, indeed, it is scarcely possible that these matters differ very widely from the picture which I have drawn of them. I venture to hope that those who enjoy finding out causes and who appreciate the wonders of light will be interested in these various speculations and in the new explanation of that remarkable property upon which the structure of the human eye depends and upon which are based those instruments which so powerfully aid the eye. I trust also there will be some who, from such beginnings, will push these investigations far in advance of what I have been able to do; for the subject is not one which is easily exhausted. This will be evident especially from those parts of the subject which I have indicated as too difficult for solution; and still more evident from those matters upon which I have not touched at all, such as the various kinds of luminous bodies and the whole question of color, which no one can yet boast of having explained.

Finally, there is much more to be learned by investigation concerning the nature of light than I have yet discovered; and I shall be greatly indebted to those who, in the future, shall furnish what is needed to complete my imperfect knowledge.

THE HAGUE, *8th of January*, 1690.

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CHAPTER I

ON THE RECTILINEAR PROPAGATION OF RAYS

DEMONSTRATIONS in optics, as in every science where geometry is applied to matter, are based upon experimental facts; as, for instance, that light travels in straight lines, that the angles of incidence and reflection are equal, and that rays of light are refracted according to the law of sines. For this last fact is now as widely known and as certainly known as either of the preceding.

Most writers upon optical subjects have been satisfied to assume these facts. But others, of a more investigating turn of mind, have tried to find the origin and the cause of these facts, considering them in themselves interesting natural phenomena. And although they have advanced some ingenious ideas, these are not such that the more intelligent readers do not still want further explanation in order to be thoroughly satisfied.

Accordingly, I here submit some considerations on this subject with the hope of elucidating, as best I may, this department of natural science, which not undeservedly has gained the reputation of being exceedingly difficult. I feel myself especially indebted to those who first began to make clear these deeply obscure matters, and to lead us to hope that they were capable of simple explanations.

But, on the other hand, I have been astonished to find these same writers accepting arguments which are far from evident as if they were conclusive and demonstrative. No one has yet given even a probable explanation of the fundamental and remarkable phenomena of light, *viz.*, why it travels in straight lines and how rays coming from an infinitude of different directions cross one another without disturbing one another.

I shall attempt, in this volume, to present in accordance with the principles of modern philosophy, some clearer and more probable reasons, first, for the rectilinear propagation of light, and, secondly, for its reflection when it meets other bodies. Later I shall explain the phenomenon of rays which are said to undergo refraction in passing through transparent bodies of different kinds. Here I shall treat also of refraction effects due to the varying density of the earth's atmosphere. Afterwards I shall examine the causes of that peculiar refraction occurring in a certain crystal which comes from Iceland. And lastly, I shall consider the different shapes required in transparent and in reflecting bodies to converge rays upon a single point or to deflect them in various ways. Here we shall see with what ease are determined, by our new theory, not only the ellipses, hyperbolas, and other curves which M. Descartes has so ingeniously devised for this purpose, but also the curve which one surface of a lens must have when the other surface is given, as spherical, plane, or of any figure whatever.

We cannot help believing that light consists in the motion of a certain material. For when we consider its production we find that here on the earth it is generally produced by fire and flame which, beyond doubt, contain bodies in a state of rapid motion, since they are able to dissolve and melt numerous other more solid bodies. And if we consider its effects, we see that when light is converged, as, for instance, by concave mirrors, it is able to produce combustion just as fire does; *i.e.*, it is able to tear bodies apart; a property that surely indicates motion, at least in the true philosophy where one believes all natural phenomena to be mechanical effects. And, in my opinion, we must admit this, or else give up all hope of ever understanding anything in physics.

Since, according to this philosophy, it is considered certain that the sensation of sight is caused only by the impulse of some form of matter upon the nerves at the base of the eye, we have here still another reason for thinking that light consists in a motion of the matter situated between us and the luminous body.

When we consider, further, the very great speed with which light is propagated in all directions, and the fact that when rays come from different directions, even those directly opposite, they cross without disturbing each other, it must be evident that we do not see luminous objects by means of matter translated from the object to us, as a shot or an arrow travels through the air. For certainly this would be in contradiction to the two properties of light which we have just mentioned, and especially to the latter. Light is then propagated in some other manner, an understanding of which we may obtain from our knowledge of the manner in which sound travels through the air.

We know that through the medium of the air, an invisible and impalpable body, sound is propagated in all directions, from the point where it is produced, by means of a motion which is communicated successively from one part of the air to another; and since this motion travels with the same speed in all directions, it must form spherical surfaces which continually enlarge until finally they strike our ear. Now there can be no doubt that light also comes from the luminous body to us by means of some motion impressed upon the matter which lies in the intervening space; for we have already seen that this cannot occur through the translation of matter from one point to the other.

If, in addition, light requires time for its passage—a point we shall presently consider—it will then follow that this motion is impressed upon the matter gradually, and hence is propagated, as that of sound, by surfaces and spherical waves. I call these *waves* because of their resemblance to those which are formed when one throws a pebble into water and which represent gradual propagation in circles, although produced by a different cause and confined to a plane surface.

As to the question of light requiring time for its propagation, let us consider first whether there is any experimental evidence to the contrary.

What we can do here on the earth with sources of light placed at great distances (although showing that light does not occupy a sensible time in passing over these distances) may be objected to on the ground that these distances are still too small, and that, therefore, we can conclude only that the propagation of light is exceedingly rapid. M. Descartes thought it instantaneous, and based his opinion upon much better evidence, furnished by the eclipse of the moon. Nevertheless, as I shall show, even this evidence is not conclusive. I shall state the matter in a manner slightly different from his in order that we may more easily arrive at all the consequences.

Let A be the position of the sun; BD a part of the orbit or annual path of the earth; ABC a straight line intersecting in C the orbit of the moon, which is represented by the circle CD. If, now, light requires time — say one hour—to traverse the space between the earth and the moon, it follows that when the earth has reached the point B, its shadow, or the interruption of light, will not yet have reached the point C, and will not reach it until one hour later. Counting from the time when the earth occupies the position B, it will be one hour later that the moon arrives at the point C and is there obscured; but this eclipse or interruption of light will not be visible at the earth until the end of still another hour. Let us suppose that during these two hours the earth has moved to the position E. From this point the moon will appear to be eclipsed at C, a position which it occupied one hour before, while the sun will be seen at A. For I assume with Copernicus that the sun is fixed and, since light travels in straight lines, must always be seen in its true position. But it is a matter of universal observation, we are told, that the eclipsed moon appears in that part of the ecliptic directly opposite the sun; while according to our view its position ought to be behind this by the angle GEC, the supplement of the angle AEC. But this is contrary to the fact, for the angle GEC will be quite easily observed, amounting to about 33°. Now according to our computation, which will be found in the memoir on the causes of the phenomena of Saturn, the distance, BA, between the earth and the sun is about 12,000 times the diameter of the earth, and consequently 400 times the distance of the moon, which is 30 diameters. The angle ECB will, therefore, be almost 400 times as great as BAE, which is 5′, *viz.*, the angular distance traversed by the earth in its orbit during an interval of two hours. Thus the angle BCE amounts to almost 33°, and likewise the angle CEG, which is 5′ greater.

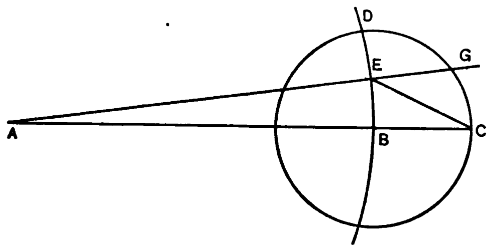


Fig. 1

But it must be noted that in this argument the speed of light is assumed to be such that the time required for it to pass from here to the moon is one hour. If, however, we suppose that it requires only a minute of time, then evidently the angle CEG will amount to only 33′; and if it requires only ten seconds of time, this angle will amount to less than 6′. But so small a quantity is not easily observed in a lunar eclipse, and consequently it is not allowable to infer the instantaneous propagation of light.

It is somewhat unusual, we must confess, to assume a speed one hundred thousand times as great as that of sound, which, according to my observations, travels about 180 toises [1151 *feet*] in a second, or during a pulse-beat; but this supposition appears by no means impossible, for it is not a question of carrying a body with such speed, but of a motion passing successively from one point to another.

I do not therefore, in thinking of these matters, hesitate to suppose that the propagation of light occupies time, for on this view all the phenomena can be explained, while on the contrary view none of them can be explained. Indeed, it seems to me, and to many others also, that M. Descartes, whose object has been to discuss all physical subjects in a clear way, and who has certainly succeeded better than any one before him, has written nothing on light and its properties which is not either full of difficulty or even inconceivable.

But this idea which I have advanced only as a hypothesis has recently been almost established as a fact by the ingenious method of Römer, whose work I propose here to describe, expecting that he himself will later give a complete confirmation of this view.

His method, like the one we have just discussed, is astronomical. He proves not only that light requires time for its propagation, but shows also how much time it requires and that its speed must be at least six times greater than the estimate which I have just given.

For this demonstration, he uses the eclipses of the small planets which revolve about Jupiter, and which very often pass into its shadow. His reasoning is as follows: Let A denote the sun; BCDE, the annual orbit of the earth; F, Jupiter; and GN, the orbit of the innermost satellite, for this one, on account of its short period, is better adapted to this investigation than is either of the other three. Let G represent the point of the satellite's entrance into, and H the point of its emergence from, Jupiter's shadow.

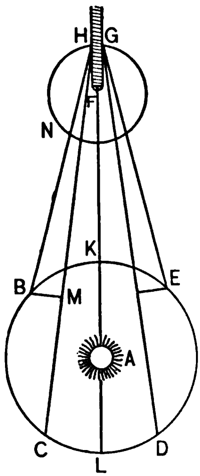


Fig. 2

Let us suppose that an emergence of this satellite has been observed while the earth occupies the position B, at some time before the last quarter. If the earth remained in this position, $42\frac{1}{2}$ hours would elapse before the next emergence would occur. For this is the time required for the satellite to make one revolution in its orbit and return to opposition with the sun. If, for instance, the earth remained at the point B during 30 revolutions, then, after an interval of 30 times $42\frac{1}{2}$ hours, the satellite would again be observed to emerge. But if meanwhile the earth has moved to a point C, more distant from Jupiter, it is evident that, provided light requires time for its propagation, the emergence of the little planet will be recorded later at C than it would have been at B. For it will be necessary to add to this interval, 30 times $42\frac{1}{2}$ hours, the time occupied by light in passing over a distance MC, the difference of the distances CH and BH. In like manner, in the other quarter, while the earth travels from D to E, approaching Jupiter, the eclipses will occur earlier when the earth is at E than if it had remained at D.

Now by means of a large number of these eclipse observations, covering a period of ten years, it is shown that these inequalities are very considerable, amounting to as much as ten minutes or more; whence it is concluded that, for traversing the whole diameter of the earth's orbit KL, twice the distance from here to the sun, light requires about 22 minutes.

The motion of Jupiter in its orbit, while the earth passes from B to C or from D to E, has been taken into account in the computation, where it is also shown that these inequalities cannot be due either to an irregularity in the motion of the satellite or to its eccentricity.

If we consider the enormous size of this diameter, KL, which I have found to be about 24 thousand times that of the earth, we get some idea of the extraordinary speed of light.

Even if we suppose that KL were only 22 thousand diameters of the earth, a speed covering this distance in 22 minutes would be equivalent to the rate of one thousand diameters per minute, *i.e.*, 16⅔ diameters a second (or a pulse-beat), which makes more than eleven hundred times one hundred thousand toises [212,222 *kilometres*], since one terrestrial diameter contains 2865 leagues, of which there are 25 to the degree, and since, according to the exact determination made by Mr. Picard in 1669 under orders from the king, each league contains 2282 toises.

But, as I have said above, sound travels at the rate of only 180 toises [350 *metres*] per second. Accordingly, the speed of light is more than 600,000 times as great as that of sound, which, however, is a very different thing from being instantaneous, the difference being exactly that between a finite quantity and infinity. The idea that luminous disturbances are handed on from point to point in a gradual manner being thus confirmed, it follows, as I have already said, that light is propagated by spherical waves, as is the case with sound.

But if they resemble each other in this respect, they differ in several others—*viz.*, in the original production of the motion which causes them, in the medium through which they travel, and in the manner in which they are transmitted in this medium.

Sound, we know, is produced by the rapid disturbance of some body (either as a whole or in part); this disturbance setting in motion the contiguous air. But luminous disturbances must arise at each point of the luminous object, else all the different parts of this object would not be visible. This fact will be more evident in what follows.

In my opinion, this motion of luminous bodies cannot be better explained than by supposing that those which are fluid, such as a flame, and apparently the sun and stars, are composed of particles that float about in a much more subtle medium, which sets them in rapid motion, causing them to strike against the still smaller particles of the surrounding ether. But in the case of luminous solids, such as red-hot metal or carbon, we may suppose this motion to be caused by the violent disturbance of the particles of the metal or of the wood, those which lie on the surface exciting the ether. Thus the motion which produces light must also be more sudden and more rapid than that which causes sound, since we do not observe that sonorous disturbances give rise to light any more than that the motion of the hand through the air gives rise to sound.

The question next arises as to the nature of the medium in which is propagated this motion produced by luminous bodies. I have called it ether; but it is evidently something different from the medium through which sound travels. For this latter is simply the air which we feel and breathe, and which, when removed from any region, leaves behind the luminiferous medium. This fact is shown by enclosing a sounding body in a glass vessel and removing the atmosphere by means of the air-pump which Mr. Boyle has devised, and with which he has performed so many beautiful experiments. But in trying this it is well to place the sounding body on cotton or feathers in such a way that it cannot communicate its vibrations either to the glass receiver or to the air-pump, a point which has hitherto been neglected. Then, when all the air has been removed, one hears no sound from the metal even when it is struck.

From this we infer not only that our atmosphere, which is unable to penetrate glass, is the medium through which sound travels, but also that it is different from that which carries luminous disturbances; for when the vessel is exhausted of air, light traverses it as freely as before.

This last point is demonstrated even more clearly by the celebrated experiment of Torricelli. That part of the glass tube which the mercury does not fill contains a high vacuum, but transmits light the same as when filled with air. This shows that there is within the tube some form of matter which is different from air, and which penetrates either glass or mercury, or both, although both the glass and the mercury are impervious to air. And if the same experiment is repeated, except that a little water be placed on top of the mercury, it becomes equally evident that the form of matter in question passes either through the glass or through the water or through both.

As to the different modes of transmission of sound and light, it is easy to understand what happens in the case of sound when one recalls that air can be compressed and reduced to a much smaller volume than it ordinarily occupies, and that just in proportion as its volume is diminished it tends to regain its original size. This property, taken in conjunction with its penetrability, which it retains in spite of compression, appears to show that it is composed of small particles which float about, in rapid motion, in an ether composed of still finer particles. Sound, then, is propagated by the effort of these air particles to escape when at any point in the path of the wave they are more compressed than at some other point.

But the enormous speed of light, together with its other properties, hardly allows us to believe that it is propagated in the same way. Accordingly, I propose to explain the manner in which I think it must occur. It will be necessary first, however, to describe that property of hard bodies in virtue of which they transmit motion from one to another.

If one takes a large number of spheres of equal size, made of any hard material, and arranges them in contact in a straight line, he will find that, on allowing a sphere of the same size to roll against one end of the line, the motion is transmitted in an instant to the other end of the line. The last sphere in the row flies off while the intermediate ones are apparently undisturbed; the sphere which originally produced the disturbance also remains at rest. Here we have a motion which is transmitted with high speed, which varies directly as the hardness of the spheres.

Nevertheless, it is certain that this motion is not instantaneous, but is gradual, requiring time. For if the motion, or, if you please, the tendency to motion, did not pass successively from one sphere to another, they would all be affected at the same instant, and would all move forward together. So far from this being the case, it is the last one only which leaves the row, and it acquires the speed of the sphere which gave the blow. Besides this experiment there are others which show that all bodies, even those which are considered hardest, such as tempered steel, glass, and agate, are really elastic, and bend to some extent whether they are made into rods, spheres, or bodies of any other shape; that is, they yield slightly at the point where they are struck, and immediately regain their original figure. For I have found that in allowing a glass or agate sphere to strike upon a large, thick, flat piece of the same material, whose surface has been dulled by the breath, the point of contact is marked by a circular disk which varies in size directly as the strength of the blow. This shows that during the encounter these materials yield and then fly back, a process which must require time.

Now to apply this kind of motion to the explanation of light, nothing prevents our imagining the particles of the ether as endowed with a hardness almost perfect and with an elasticity as great as we please. It is not necessary here to discuss the cause either of this hardness or of this elasticity, for such a consideration would lead us too far from the subject. I will, however, remark in passing that these ether particles, in spite of their small size, are in turn composed of parts, and that their elasticity consists in a very rapid motion of a subtle material which traverses them in all directions and compels them to assume a structure which offers an easy and open passage to this fluid. This accords with the theory of M. Descartes, except that I do not agree with him in assigning to the pores the form of hollow circular canals. So far from there being anything absurd or impossible in all this, it is quite credible that nature employs an infinite series of different-sized molecules, endowed with different velocities, to produce her marvellous effects.

But although we do not understand the cause of elasticity, we cannot fail to observe that most bodies possess this property: it is not unnatural, therefore, to suppose that it is a characteristic also of the small, invisible particles of the ether. If, indeed, one looks for some other mode of accounting for the gradual propagation of light, he will have difficulty in finding one better adapted than elasticity to explain the fact of uniform speed. And this appears to be necessary; for if the motion slowed up as it became distributed through a larger mass of matter, and receded farther from the source of light, then its high speed would be lost at great distances. But we suppose the elasticity to be a property of the ether so that its particles regain their shape with equal rapidity whether they are struck with a hard or a gentle blow; and thus the rate at which the light moves remains the same [*at all distances from the source*].

Nor is it necessary that the ether particles should be arranged in straight lines, as was the case with our row of spheres. The most irregular configuration, provided the particles are in contact with each other, will not prevent their transmitting the motion and handing it on to the regions in front. It is to be noted that we have here a law of motion which governs this kind of propagation, and which is verified by experiment, *viz.*, when a sphere such as A, touching several other smaller ones, CCC, is struck by another sphere, B, in such a way as to make an impression upon each of its neighbors, it transfers its motion to them and remains at rest, as does also the sphere B. Now, without supposing that ether particles are spherical (for I do not see that this is necessary), we can nevertheless understand that this law of impulses plays a part in the propagation of the motion.

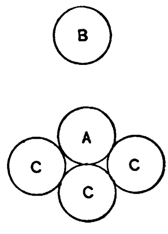


Fig. 3

Equality of size would appear to be a more necessary assumption, since otherwise we should expect the motion to be reflected on passing from a smaller to a larger particle, following the laws of percussion which I published some years ago. Yet, as will appear later, this equality is necessary not so much to make the propagation of light possible as to make it easy and intense. Nor does it appear improbable that the ether particles were made equal for a purpose so important as the transmission of light. This may be true, at least, in the vast region lying beyond our atmosphere and serving only to transmit the light of the sun and the stars.

I have now shown how we may consider light as propagated, in time, by spherical waves, and how it is possible that the speed of propagation should be as great as that demanded by experiment and by astronomical observation. It must, however, be added that although the ether particles are supposed to be in continual motion (and there is much evidence for this view), the gradual transmission of the waves is not thus interfered with. For it does not consist in a translation of these particles, but merely in a small vibration, which they are compelled to transmit to their neighbors in spite of their proper motion and their change of relative position.

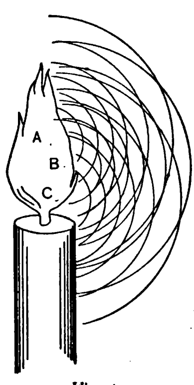


Fig. 4

But we must consider, in greater detail, the origin of these waves and the manner of their propagation from one point to another. And, first, it follows from what has already been said concerning the production of light that each point of a luminous body, such as the sun, a candle, or a piece of burning carbon, gives rise to its own waves, and is the centre of these waves. Thus if A, B, and C represent different points in a candle flame, concentric circles described about each of these points will represent the waves to which they give rise. And the same is true for all the points on the surface and within the flame. But since the disturbances at the centre of these waves do not follow each other in regular succession, we need not imagine the waves to follow one another at equal intervals; and if, in the figure, these waves are equally spaced, it is rather to indicate the progress which one and the same wave has made during equal intervals of time than to represent several waves having their origin at the same point.*

Nor does this enormous number of waves, crossing one another without confusion and without disturbing one another, appear unreasonable, for it is well known that one and the same particle of matter is able to transmit several waves coming from different, and even opposite, directions. And this is true not only in the case where the displacements follow one another in succession, but also where they are simultaneous. This is because the motion is propagated gradually. It is shown by the row of hard and equal spheres above mentioned. If we allow two equal spheres, A and D, to strike against the opposite sides of this row at the same instant, they will be observed to rebound each with the same speed that it had before collision, while all the other spheres remain at rest, although the motion has twice traversed the entire row. [*This evidently implies that the spheres A and D have equal speeds just before*

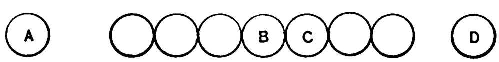


Fig. 5

collision.] If these two oppositely directed motions happen to meet at the middle sphere, B, or at any other sphere, say C, it will yield and spring back from both sides, thus transmitting both motions at the same instant.

* [*From this paragraph it would appear that Huygens had no conception of* trains *of light-waves. The experimental evidence for thinking that light-waves travel in trains seems first to have been furnished by Young. See pp.* 60, 61 *below. If, however, one prefers to interpret the colored rings of Newton in terms of the wave-theory, this experimental evidence may be ascribed to Newton.*]

But what is strangest and most astonishing of all is that waves produced by displacements and particles so minute should spread to distances so immense, as, for instance, from the sun or from the stars to the earth. For the intensity of these waves must diminish in proportion to their distance from the origin until finally each individual wave is of itself unable to produce the sensation of light. Our astonishment, however, diminishes when we consider that in the great distance which separates us from the luminous body there is an infinitude of waves which, although coming from different parts of the [*luminous*] body, are practically compounded into a single wave which thus acquires sufficient intensity to affect our senses. Thus the infinitely great number of waves which at any one instant leave a fixed star, as large possibly as our sun, unite to form what is equivalent to one single wave of intensity sufficient to affect the eye. Not only so, but each luminous point may send us thousands of waves in the shortest imaginable time, on account of the rapidity of the blows with which the particles of the luminous body strike the ether at these points. The effect of the waves would thus be rendered still more sensible.

In considering the propagation of waves, we must remember that each particle of the medium through which the wave spreads doe not communicate its motion only to that neighbor which lies in the straight line drawn from the luminous point, but shares it with all the particles which touch it and resist its motion. Each particle is thus to be considered as the centre of a wave. Thus if DCF is a wave whose centre and origin is the luminous point A, a particle at B, inside the sphere DCF, will give rise to its own individual [*secondary*] wave, KCL, which will touch the wave DCF in the point C, at the same instant in which the principal wave, originating at A, reaches the position DCF. And it is clear that there will be only one point of the wave KCL which will touch the wave DCF, *viz.*, the point which lies in the straight line from A

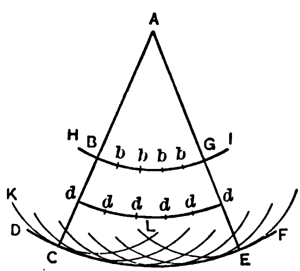


Fig. 6

drawn through B. In like manner, each of the other particles, *bbbb*, etc., lying within the sphere DCF, gives rise to its own wave. The intensity of each of these waves may, however, be infinitesimal compared with that of DCF, which is the resultant of all those parts of the other waves which are at a maximum distance from the centre A.

We see, moreover, that the wave DCF is determined by the extreme limit to which the motion has travelled from the point A within a certain interval of time. For there is no motion beyond this wave, whatever may have been produced inside by those parts of the secondary waves which do not touch the sphere DCF. Let no one think this discussion mere hair-splitting. For, as the sequel will show, this principle, so far from being an ultra-refinement, is the chief element in the explanation of all the properties of light, including the phenomena of reflection and refraction. This is exactly the point which seems to have escaped the attention of those who first took up the study of light-waves, among whom are Mr. Hooke, in his *Micrographia,* and Father Pardies, who had undertaken to explain reflection and refraction on the wave-theory, as I know from his having shown me a part of a memoir which he was unable to finish before his death. But the most important fundamental idea, which consists in the principle I have just stated, is wanting in his demonstrations. On other points also his view is different from mine, as will some day appear in case his writings have been preserved.

Passing now to the properties of light, we observe first that each part of the wave is propagated in such a way that its extremities lie always between the same straight lines drawn from the luminous point.

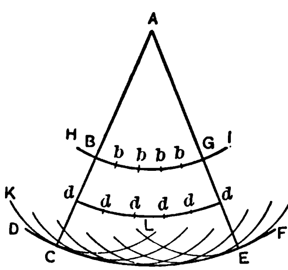


Fig. 6

For instance, that part of the wave BG, whose centre is the luminous point A, develops into the arc CE, limited by the straight lines, ABC and AGE. For although the secondary waves produced by the particles lying within the space CAE may spread to the region outside, nevertheless they do not combine at the same instant to produce one single wave limiting the motion and lying in the circumference CE which is their common tangent. This explains the fact that light, provided its rays are not reflected or refracted, always travels in straight lines, so that no body is illuminated by it unless the straight-line path from the source to this body is unobstructed.

Let us, for instance, consider the aperture BG as limited by the opaque bodies BH, GI; then, as we have just indicated, the light-waves will always be limited by the straight lines AC, AE. The secondary waves which spread into the region outside of ACE have not sufficient intensity to produce the sensation of light.

Now, however small we may make the opening BG, the circumstances which compel the light to travel in straight lines still remain the same; for this aperture is always sufficiently large to contain a great number of these exceedingly minute ether particles. It is thus evident that each particular part of any wave can advance only along the straight line drawn to it from the luminous point. And this justifies us in considering rays of light as straight lines.

From what has been said concerning the small intensity of the secondary waves, it would appear not to be necessary that all the ether particles be equal, although such an equality would favor the propagation of the motion. The effect of inequality would be to make a particle, in colliding with a larger one, use up a part of its momentum in an effort to recover. The secondary waves thus sent backward towards the luminous point would be unable to produce the sensation of light, and would not result in a primary wave similar to CE.

Another and more remarkable property of light is that when rays come from different, or even opposite, directions each produces its effect without disturbance from the other. Thus several observers are able, all at the same time, to look at different objects through one single opening; and two individuals can look into each other's eyes at the same instant.

If we now recall our explanation of the action of light and of waves crossing without destroying or interrupting each other, these effects which we have just described are readily understood, though they are not so easily explained from Descartes' point of view, *viz.*, that light consists in a continuous [*hydrostatic*] pressure which produces only a tendency to motion.

For such a pressure cannot, at the same instant, affect bodies from two opposite sides unless these bodies have some tendency to approach each other. It is, therefore, impossible to understand how two persons can look each other in the eye or how one torch can illuminate another.

CHAPTER II

ON REFLECTION

HAVING explained the effects produced by light-waves in a homogeneous medium, we shall next consider what happens when they impinge upon other bodies. First of all we shall see how reflection is explained by these same waves and how the equality of angles follows as a consequence. Let AB represent a plane polished surface of some metal, glass, or other substance, which, for the present, we shall consider as perfectly smooth (concerning irregularities which are unavoidable we shall have something to say at the close of this demonstration); and let the line AC, inclined to AB, represent a part of a light-wave whose centre is so far away that this part AC may be considered as a straight line. It may be mentioned here, once for all, that we shall limit our consideration to a single plane, *viz.*, the plane of the figure, which passes through the centre of the spherical wave and cuts the plane AB at right angles.

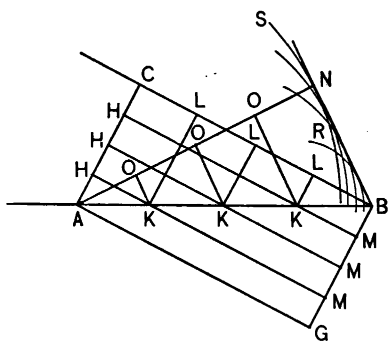


Fig. 7

The region immediately about C on the wave AC will, after a certain interval of time, reach the point B in the plane AB, travelling along the straight line CB, which we may think of as drawn from the source of light and hence drawn perpendicular to AC. Now in this same interval of time the region about A on the same wave is unable to transmit its entire motion beyond the plane AB; it must, therefore, continue its motion on this side of the plane to a distance equal to CB, sending out a secondary spherical wave in the manner described above. This secondary wave is here represented by the circle SNR, drawn with its centre at A and with its radius AN equal to CB.

So, also, if we consider in turn the remaining parts H of the wave AC, it will be seen that they not only reach the surface AB along the straight lines HK parallel to CB, but they will produce, at the centres K, their own spherical waves in the transparent medium. These secondary waves are here represented by circles whose radii are equal to KM—that is, equal to the prolongations of HK to the straight line BG which is drawn parallel to AC. But, as is easily seen, all these circles have a common tangent in the straight line BN, *viz.*, the same line which passes through B and is tangent to the first circle having A as centre and AN, equal to BC, as radius.

This line BN (lying between B and the point N, the foot of the perpendicular let fall from A) is the envelope of all these circles, and marks the limit of the motion produced by the reflection of the wave AC. It is here that the motion is more intense than at any other point, because, as has been explained, BN is the new position which the wave AC has assumed at the instant when the point C has reached B. For there is no other line which, like BN, is a common tangent to these circles, unless it be BG, on the other side of the plane AB. And BG will represent the transmitted wave only in case the motion occurs in a medium which is homogeneous with that above the plane. If, however, one wishes to see just how the wave AC has gradually passed into the wave BN, he has only to use the same figure and draw the straight lines KO parallel to BN, and the straight lines KL parallel to AC. It is thus seen that the wave AC, from being a straight line, passes

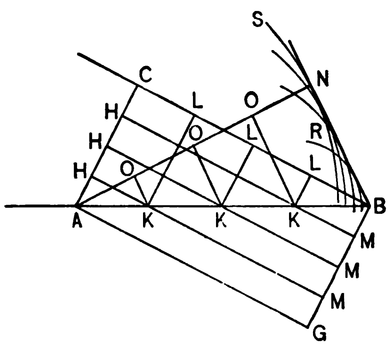


Fig. 7

successively into all the broken lines OKL, and reassumes the form of a single straight line NB.

It is now evident that the angle of reflection is equal to the angle of incidence. For the right-angled triangles ABC and BNA have the side AB in common, and the side CB equal to the side NA, whence it follows that the angles opposite these sides are equal, and hence also the angles CBA and NAB. But CB, perpendicular to CA, is the direction of the incident ray, while AN, perpendicular to the wave BN, has the direction of the reflected ray. These rays are, therefore, equally inclined to the plane AB.

Against this demonstration it may be urged that while BN is the common tangent of the circular waves in the plane of this figure, the fact is that these waves are really spherical and have an infinitely great number of similar tangents, *viz.*, all straight lines drawn through the point B and lying in the surface of a cone generated by the revolution of a straight line BN about BA as axis. It remains to be shown, therefore, that this objection presents no difficulty; and, incidentally, we shall see that the incident and reflected rays each lie in one plane perpendicular to the reflecting plane.

I remark, then, that the wave AC, so long as it is considered merely a line, can produce no light. For a ray of light, however slender, must have a finite thickness in order to be visible. In order, therefore, to represent a wave whose path is along this ray, it is necessary to replace the straight line AC by a plane area, as, for instance, by the circle HC in the following figure, where the luminous point is supposed to be infinitely distant. From the preceding proof it is easily seen that each element of area on the wave HC, having reached the plane AB, will there give rise to its own secondary wave; and when C reaches the point B, these will all have a common tangent plane, *viz.*, the circle BN equal to CH. This circle will be cut through the centre and at right angles by the same plane which thus cuts the circle CH and the ellipse AB.

It is thus seen that the spherical secondary waves can have no common tangent plane other than BN. In this plane will be located more of the reflected motion than in any other, and it will therefore receive the light transmitted from the wave CH.

I have noted in the preceding explanation that the motion of the wave incident at A is not transmitted beyond the plane AB, at least not entirely. And here it is necessary to remark that, although the motion of the ether may be partly communicated to the reflecting body, this cannot in the slightest alter the speed of the propagation of the waves, which determines the angle of reflection. For, in any one medium, a slight disturbance produces waves which travel with the same speed as those

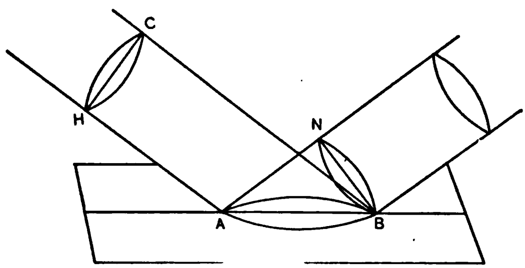


Fig. 8

due to a very great disturbance, a consequence of that property of elastic bodies concerning which we have spoken above, *viz.*, the time occupied in recovery is the same whether the compression be large or small. In every case of reflection of light from the surface of any substance whatever the angles of incidence and reflection are therefore equal, even though the body be of such a nature as to absorb a part of the motion delivered by the incident wave. And, indeed, experiment shows that among polished bodies there is no exception to this law of reflection.

We must emphasize the fact that in our demonstration there is no need that the reflecting surface be considered a perfectly smooth plane, as has been assumed by all those who have attempted to explain the phenomena of reflection. All that is called for is a degree of smoothness such as would be produced by the particles of the reflecting medium being placed one near another. These particles are much larger than those of the ether, as will be shown later when we come to treat of the transparency and opacity of bodies. Since, now, the surface consists thus of particles assembled together, the ether particles being above and smaller, it is evident that one cannot demonstrate the equality of the angles of incidence and reflection from the time-worn analogy with that which happens when a ball is thrown against a wall. By our method, on the other hand, the fact is explained without difficulty.

Take particles of mercury, for instance, for they are so small that we must think of the least visible portion of surface as containing millions, arranged like the grains in a heap of sand which one has smoothed out as much as possible; this surface for our purpose is equal to polished glass. And, though such a surface may be always rough compared with ether particles, it is evident that the centres of all the secondary waves of reflection which we have described above lie practically in one plane. Accordingly, a single tangent comes as near touching them all as is necessary for the production of light. And this is all that is required in our demonstration to explain the equality of angles without allowing the rest of the motion, reflected in various directions, to produce any disturbing effect.

CHAPTER III

ON REFRACTION

In the same manner that reflection has been explained by light-waves reflected at the surface of polished bodies, we propose now to explain transparency and the phenomena of refraction by means of waves propagated into and through transparent bodies, whether solids, such as glass, or liquids, such as water and oils. But, lest the passage of waves into these bodies appear an unwarranted assumption, I will first show that this is possible in more ways than one.

Let us imagine that the ether does penetrate any transparent body, its particles will still be able to transmit the motion of the waves just as do those of the ether, supposing them each to be elastic. And this we can easily believe to be the case with water and other transparent liquids, since they are composed of discrete particles. But it may appear more difficult in the case of glass and other bodies that are transparent and hard, because their solidity would hardly allow that they should take up any motion except that of their mass as a whole. This, however, is not necessary, since this solidity is not what it appears to us to be, for it is more probable that these bodies are composed of particles which are placed near one another and bound together by an external pressure due to some other kind of matter and by irregularity of their own configurations. For their looseness of structure is seen in the facility with which they are penetrated by the medium of magnetic vortices and those which cause gravitation. One cannot go further than to say that these bodies have a structure similar to that of a sponge, or of light bread, because heat will melt them and change the relative positions of their particles. We infer, then, as has been indicated above, that they are assemblages of particles touching one another but not forming a continuous solid. This being the case, the motion which these particles receive in the transmission of light is simply communicated from one to another, while the particles themselves remain tethered in their own positions and do not become disarranged among themselves. It is easily possible for this to occur without in any way affecting the solidity of the structure as seen by us.

By the external pressure of which I have spoken is not to be understood that of the air, which would be quite insufficient, but that of another and more subtle medium, whose pressure is exhibited by an experiment which I chanced upon a long while ago, namely, that water from which the air has been removed remains suspended in a glass tube open at the lower end, even though the air may have been exhausted from a vessel enclosing this tube.

We may thus explain transparency without assuming that bodies are penetrated by the luminiferous ether or that they contain pores through which the ether can pass. The fact, however, is not only that this medium penetrates ordinary bodies, but that it does so with the utmost ease, as indeed has already been shown by the experiment of Torricelli which we have cited above. When the mercury or the water leaves the upper part of the glass tube, the ether appears at once to take its place and transmit light. But following is still another argument for thinking that bodies, not only those which are transparent, but others also, are easily penetrable.

When light traverses a hollow glass sphere which is completely closed, it is evident that the sphere is filled with ether, just as is the space outside the sphere. And this ether, as we have shown above, consists of particles lying in close contact with each other. If, now, it were enclosed in the sphere in such a way that it could not escape through the pores of the glass, it would be compelled to partake of any motion which one might impress upon the sphere; consequently nearly the same force would be required to impress a given speed upon this sphere, lying upon a horizontal plane, as if it were filled with water, or possibly mercury. For the resistance which a body offers to any velocity one may wish to impart to it varies directly as the quantity of matter which the body contains and which is compelled to acquire velocity. But the fact is that the sphere resists the motion only in proportion to the amount of glass in it. Whence it follows that the ether within is not enclosed, but flows through the glass with perfect freedom.

Later we shall show, by this same process, that penetrability may be inferred for opaque bodies also.

A second and more probable explanation of transparency is to say that the light-waves continue on in the ether which always fills the interstices, or pores, of transparent bodies. For since it passes continuously and with ease, it follows that these pores are always full. Indeed, it may be shown that these interstices occupy more space than the particles which make up the body.

Now if it be true, as we have said, that the force required to impart a given horizontal velocity to a body is proportional to the mass of the body, and if this force be also proportional to the weight of the body, as we know by experiment that it is, then the mass of any body must be also proportional to its weight. Now we know that water weighs only $\frac{1}{14}$ part as much as an equal volume of mercury, therefore the substance of the water occupies only $\frac{1}{14}$ part of the space that encloses its mass. Indeed, it must occupy even a smaller fraction than this, because mercury is not so heavy as gold, and gold is a substance which is not very dense, since the medium of magnetic vortices and that which causes gravitation penetrate it with the utmost ease.

But it may be objected that if water be a substance of such small density, and if its particles occupy so small a portion of its apparent volume, it is very remarkable that it should offer such stubborn resistance to compression; for it has not been condensed by any force hitherto employed, and remains perfectly liquid while under pressure.

This is, indeed, no small difficulty. But it may nevertheless be explained by supposing that the very violent and rapid motion of the subtle medium which keeps water liquid also sets in motion the particles of which it is composed, and maintains this liquid state in spite of any pressure which has hitherto been applied.

If, now, the structure of transparent bodies be as loose as we have indicated, we may easily imagine waves penetrating the ether which fills the interstices between the particles. Not only so, but we can easily believe that the speed of these waves inside the body must be a little less on account of the small détours necessitated by these same particles. I propose to show that in this varying velocity of light lies the cause of refraction.

I will first indicate a third and last method by which we may explain transparency, namely, by supposing that the motion of the light-waves is transmitted equally well by the ether particles which fill the interstices of the body, and by the particles which compose the body, the motion being handed on from one to the other. A little later we shall see how beautifully this hypothesis explains the double refraction of certain transparent substances. Should one object that the particles of ether are much smaller than those of the transparent body, since the former pass through the intervals between the latter, and that consequently they would be able to communicate only a small portion of their momentum, we may reply that the particles of the body are composed of other still smaller particles, and that it is these secondary particles that take up the momentum from those of the ether.

Moreover, if the particles of the transparent body are slightly less elastic than are the ether particles, which we may reasonably suppose, it would still follow that the speed of the light waves is less inside the body than outside in the ether.

We have here, what appears to me, the manner in which light-waves are probably transmitted by transparent bodies. But there still remains the consideration of opaque bodies and the difference between these and transparent bodies, a question all the more interesting in view of the ease with which ether penetrates all bodies, a fact to which attention has already been directed, and which might lead one to think that all bodies should be transparent. For considering the hollow sphere, by which I have shown the open structure of glass and the ease with which ether passes through it, one would naturally infer the same penetrability as a property of metals and all other substances. Imagine the sphere to be of silver; it would then certainly contain luminiferous ether, because this substance, as well as air, would be present in it when the opening in the sphere was closed up. But when closed and placed upon a horizontal plane it would resist motion only in proportion to the amount of silver in it, showing as above that the enclosed ether does not acquire the motion of the sphere. Silver, therefore, like glass, is easily penetrated by ether. In between the particles of silver and of all other opaque bodies this substance is distributed continuously and abundantly; and, since it can transmit light, we are led to expect that these bodies should be as transparent as glass, which, however, is not the fact.

How, then, shall we explain their opacity? Are their constituent particles soft and built up of still smaller particles, and thus able to change shape when they are struck by ether particles? Do they thus damp out the motion and stop the propagation of the light-waves? This seems hardly possible; for if the particles of a metal were soft, how could polished silver and mercury reflect light so well? What seems to me more probable is that metallic bodies, which are almost the only ones that are really opaque, have interspersed among their hard particles some which are soft, the former producing reflection, the latter destroying transparency; while, on the other hand, transparent bodies are made up of only hard and elastic particles, which, together with the ether, propagate light-waves in the manner already indicated.

We pass now to the explanation of refraction, assuming, as above, that light-waves pass through transparent substances and in them undergo diminution of speed.

The fundamental phenomenon in refraction is the following, *viz.*, when any ray of light, AB, travelling in air, strikes obliquely upon the polished surface of a transparent body, FG, it undergoes a sudden change of direction at the point of incidence, B; and this change occurs in such a way that the angle CBE, which the ray makes with the normal to the surface, is less than the angle ABD, which the ray in air made with the same normal. To determine the numerical value of these angles, describe about the point B a circle cutting the rays AB, BC. Then the perpendiculars, AD, CE, let fall from these points of intersection upon the normal, DE, *viz.*, the sines of the angles ABD, CBE, bear to one another a certain ratio which, for any one transparent body, is constant for all directions of the incident ray. For glass this ratio is almost exactly $\frac{3}{2}$, while for water it is very nearly $\frac{4}{3}$, thus varying from one transparent body to another.

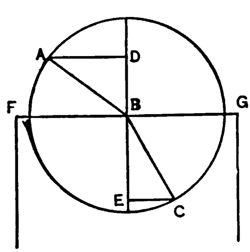


Fig. 9

Another property, not unlike the preceding, is that the refractions of rays entering and of rays emerging from a transparent body are reciprocal. That is to say, if an incident ray, AB, be refracted by a transparent body into the ray BC, so also will a ray, CB, in the interior of the body be refracted, on emergence, into the ray BA.

In order to explain these phenomena on our theory, let the straight line AB Fig. 10, represent the plane surface bounding a transparent body extending in a direction between C and N.

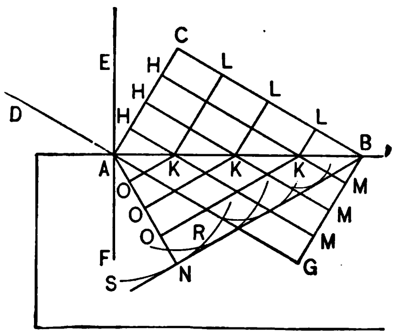


Fig. 10

By the use of the word plane we do not mean to imply a perfectly smooth surface, but merely such a one as was employed in treating of reflection, and for the same reason. Let the line AC represent a part of a light-wave whose source is so distant that this part may be treated as a straight line. The region C, on the wave AC, will, after a certain interval of time, arrive at the plane AB, along the straight line CB, which we must think of as drawn from the source of light, and which will, therefore, intersect AC at right angles. But during this same interval of time the region about A would have arrived at G, along the straight line AG, equal and parallel to CB, and, indeed, the whole of the wave AC would have reached the position GB, provided the transparent body were capable of transmitting waves as rapidly as the ether. But suppose that the rate of transmission is less rapid, say one-third less. Then the motion from the point A will extend into the transparent body to a distance which is only two-thirds of CB, while producing its secondary spherical wave as described above. This wave is represented by the circle SNR, whose centre is at A and whose radius is equal to $\frac{2}{3}$ CB. If we consider, in like manner, the other points H of the wave AC, it will be seen that, during the same time which C employs in going to B, these points will not only have reached the surface AB, along the straight lines HK, parallel to CB, but they will have started secondary waves into the transparent body from the points K as centres. These secondary waves are represented by circles whose radii are respectively equal to $\frac{2}{3}$ of the distances KM—that is, $\frac{2}{3}$ of the prolongations of HK to the straight line BG. If the two transparent media had the same ability to transmit light, these radii would equal the whole lengths of the various lines KM.

But all these circles have a common tangent in the line BN, *viz.*, the same line which we drew from the point B tangent to the circle SNR first considered. For it is easy to see that all the other circles from B up to the point of contact, N, touch, in the same manner, the line BN, where N is also the foot of the perpendicular let fall from A upon BN.

We may, therefore, say that BN is made up of small arcs of these circles, and that it marks the limits which the motion from the wave AC has reached in the transparent medium, and the region where this motion is much greater than anywhere else. And, furthermore, that this line, as already indicated, is the position assumed by the wave AC at the instant when the region C has reached the point B. For there is no other line below the plane AB, which, like BN, is a common tangent to all these secondary waves.

Accordingly, if one wishes to discover through what intermediate steps the wave AC reached the position BN, he has only to draw, in the same figure, the straight lines KO parallel to BN, and all the lines KL parallel to AC. He will thus see that the wave CA passes from a straight line into the successive broken lines LKO, reassuming the form of a straight line in the position BN. From what has preceded this will be so evident as to need no further explanation.

If, now, using the same figure, we draw EAF normal to the plane AB at the point A, and draw DA at right angles to the wave AC, the incident ray of light will then be represented by DA; and AN, which is drawn perpendicular to BN, will be the refracted ray; for these rays are merely the straight lines along which the parts of the waves travel.

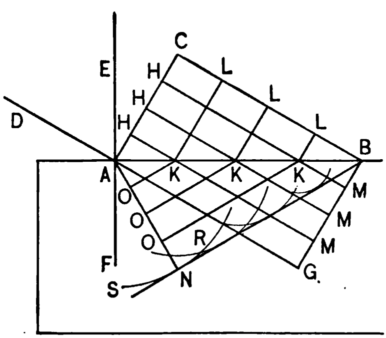


Fig. 10

From the foregoing it is easy to deduce the principal law of refraction, *viz.*, that the sine of the angle DAE always bears a constant ratio of the sine of the angle NAF, whatever may be the direction of the incident ray, and that the ratio is the same as that which the speed of the waves in the medium on the side AE bears to their speed on the side AF.

For if we consider AB as the radius of a circle, the sine of the angle BAC is BC, and the sine of the angle ABN is AN. But the angles BAC and DAE are equal; for each is the complement of CAE. And the angle ABN is equal to NAF, since each is the complement of BAN. Hence the sine of the angle DAE is to the sine NAF as BC is to AN. But the ratio of BC to AN is the same as that of the speeds of light in the media on the side towards AE and the side towards AF, respectively; hence, also, the sine of the angle DAE bears to the sine of the angle NAF the same ratio as these two speeds of light.

In order to follow the refracted ray when the light-waves enter a body which transmits them more rapidly than the body from which they emerge (say in the ratio of 3 to 2), it is necessary only to repeat the same construction and the same demonstration which we have just been using, substituting, however, $\frac{3}{2}$ in place of $\frac{2}{3}$. And we find, by the same logical process, employing this other figure, that when the region C of the wave AC reaches the point B of the surface AB, the whole wave AC will have advanced to the position BN, such that the ratio of BC, perpendicular to AC, is to AN, perpendicular to BN, as 2 is to 3. The same ratio will also hold between the sine of the angle EAD and the sine of the angle FAN.

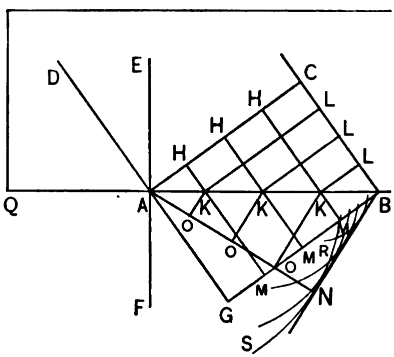


Fig. 11

The reciprocal relations between the refractions of a ray on entering and on emerging from one and the same medium is thus made evident. If the ray NA is incident upon the exterior surface AB, and is refracted into AD, then the ray DA on emerging from the medium will be refracted into AN.

We are now able to explain a remarkable phenomenon which occurs in this refraction. When the incident ray DA exceeds a certain inclination it loses its ability to pass into the other medium. Because if the angle DAQ or CBA is such that, in the triangle ACB, CB is equal to or greater than $\frac{2}{3}$ of AB, then AN, being equal to or greater than AB, can no longer form one side of the triangle ANB. Therefore the wave BN does not exist, and consequently there can be no line AN drawn at right angles to it. And thus the incident ray DA cannot penetrate the surface AB.

When the wave-speeds are in the ratio of 2 to 3, as in the case of glass and air, which we have considered, the angle DAQ must exceed 48° 11′ if the ray DA is to emerge. And when the ratio of speeds is that of 3 to 4, as is almost exactly the case in water and air, this angle DAQ must be greater than 41° 24′. And this agrees perfectly with experiment.

But one may here ask why no light penetrates the surface, since the encounter of the wave AC against the surface AB must produce some motion in the medium on the other side. The answer is simple, if we recall what has already been said. For although an infinite number of secondary waves may be started into the medium on the other side of AB, these waves at no time have a common tangent line, either straight or curved. There is thus no line which marks the limit to which the wave AC has been transmitted beyond the plane AB, nor is there any line in which the motion has been sufficiently concentrated to produce light.

In the following manner one may easily recognize the fact that, when CB is greater than $\frac{2}{3}$ AB, the waves beyond the plane AB have no common tangent. About the centres K describe circles having radii respectively equal to $\frac{3}{2}$ LB. These circles will enclose one another and will each pass beyond the point B.

It is to be noted that just as soon as the angle DAQ becomes too small to allow the refracted ray DA to pass into the other medium, the internal reflection which occurs at the surface AB increases rapidly in brilliancy, as may be easily shown by means of a triangular prism. In terms of our theory, we may thus explain this phenomenon: While the angle DAQ is still large enough for the ray DA to be transmitted, it is evident that the light from the wave-front AC will be concentrated into a much shorter line when it reaches the position BN. It will be seen also that the wave-front BN grows shorter in proportion as the angle CBA or DAQ becomes smaller, until finally, when the limit indicated above is reached, BN is reduced to a point. That is to say, when the region about C, on the wave AC, reaches B, the wave BN, which is the wave AC after transmission, is entirely compressed into this same point B ; and, in like manner, when the region about H has reached the point K the part AH is completely reduced to this same point K. It follows, therefore, that in proportion as the direction of propagation of the wave AC happens to coincide with the surface AB, so will be the quantity of motion along this surface.

Now this motion must necessarily spread into the transparent body and also greatly reinforce the secondary waves which produce internal reflection at the face AB, according to the laws of this reflection explained above.

And since a small diminution in the angle of incidence is sufficient to reduce the wave-front BN from a fairly large quantity to zero (for if this angle in the case of glass be 49° 11′, the angle BAN amounts to as much as 11° 21′; but if this same angle DAQ be diminished by one degree only, the angle BAN becomes zero and the wave-front BN is reduced to a point), it follows that the internal reflection occurs suddenly, passing from comparative darkness to brilliancy at the instant when the angle of incidence assumes a value which no longer permits refraction.

Now as to ordinary external reflection, *i. e.*, reflection which occurs when the angle of incidence DAQ is still large enough to allow the refracted ray to pass through the face AB, this reflection must be from the particles which bound the transparent body on the outside, apparently from particles of air and from others which are mixed with, but are larger than, the ether particles.

On the other hand, external reflection from bodies is produced by the particles which compose these bodies, and which are larger than those of the ether, since the ether flows through the interstices of the body.

It must be confessed that we here find difficulty in explaining the experimental fact that internal reflection occurs even where the particles of air can cut no figure, as, for instance, in vessels and tubes from which the air has been exhausted.

Experiment shows further that these two reflections are of almost equal intensity, and that in various transparent bodies this intensity increases directly as the refractive index. Thus we see that reflection from glass is stronger than that from water, while in turn that from diamond is stronger than that from glass.

I shall conclude this theory of refraction by demonstrating a remarkable proposition depending upon it, namely, that when a ray of light passes from one point to another, the two points lying in different media, refraction at the bounding surface occurs in such a way as to make the time required the least possible; and exactly the same thing occurs in reflection at a plane surface. M. Fermat discovered this property of refraction, believing with us and in opposition to M. Descartes that light travels more slowly through glass than through air. But, besides this, he assumed what we have just proved from the fact that the velocities in the two media are different, *viz.*, that the ratio of the sines is a constant; or, what amounts to the same thing, he assumed, besides the different velocities, that the time employed was a minimum; and from this he derived the constancy of the sine ratio.

His demonstration, which may be found in his works and in the correspondence of M. Descartes, is very long. It is for this reason that I here offer a simpler and easier one.

Let KF represent a plane surface; imagine the point A in the medium through which the light travels more rapidly, say air; the point C lies in another, say water, in which the speed of light is less. Let us suppose that a ray passes from A, through B, to C, suffering refraction at B, according to the law above demonstrated; or, what is the same thing, having drawn PBQ perpendicular to the surface, the sine of the angle ABP is to the sine of the angle CBQ in the same ratio as the speed of light in the medium

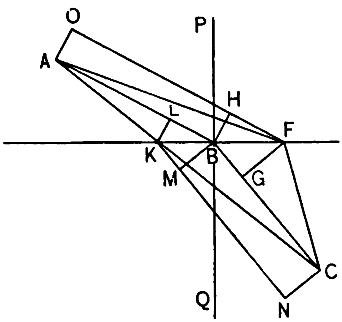


Fig. 12

containing A is to the speed in the medium containing C. It remains to show that the time required for the light to traverse AB and BC taken together is the least possible. Let us assume that the light takes some other path, say AF, FC, where F, the point at which refraction occurs, is more distant than B from A. Draw AO perpendicular to AB, and FO parallel to BA; BH perpendicular to FO, and FG perpendicular to BC. Since, now, the angle HBF is equal to PBA, and the angle BFG is equal to QBC, it follows that the sine of the angle HBF will bear to the sine of BFG the same ratio as the speed of light in the medium A bears to the speed in the medium C. But if we consider BF the radius of a circle, then sines are represented by the lines HF, BG. Accordingly, the lines HF, BG are in the ratio of these speeds. If, therefore, we imagine OF to be the incident ray, the time of passage from H to F will be the same as the time of passage from B to G in the medium C.

But the time from A to B is equal to the time from O to H. Hence the time from O to F is the same as the time from A to G, *via* B. Again, the time along FC is greater than the time along GC; and hence the time along the route OFC is greater than that along the path ABC. But AF is greater than OF; hence, *a fortiori*, the time along AFC is greater than that along ABC.

Let us now assume that the ray passes from A to C by the route AK, KC, the point of refraction, K, being nearer to A than is B. Draw CN perpendicular to BC; KN parallel to BC; BM perpendicular to KN; and KL perpendicular to BA.

Here BL and KM represent the sines of the angles BKL and KBM—that is, the angles PBA and QBC; and hence they are in the same ratio as the speeds of light in the media A and C respectively. The time, therefore, from L to B is equal to the time from K to M; and, since the time from B to C is equal to the time from M to N, the time by the path LBC is the same as the time *via* KMN. But the time from A to K is greater than the time from A to L, and, therefore, the time by the route AKN is greater than the route ABC.

Not only so, but since KC is greater than KN, the time by the path AKC will be so much the greater than by the path ABC. Hence follows that which was to be proved, namely, that the time along the path ABC is the least possible.

MEMOIRS ON

BIOGRAPHICAL SKETCH

WHILE there are no sharp lines in nature, there is a very true sense in which the year 1642, marking the death of Galileo and the birth of Newton, serves as a line of demarcation between the foundation and the superstructure of modern physics.

Galileo, by his careful study of gravitation, by his clear grasp of force as determining acceleration, by his careful search after causes and their respective effects, by his profound faith in experiment, had more than cleared the ground for the builders of modern physics. The rapid rise of this structure at the hands of Newton and his brilliant contemporaries, Boyle, Leibnitz, Römer, Du Fay, Bradley, and Hooke, marks a distinctly modern era compared with that of Galileo.

The work of Christiaan Huygens, the "Dutch Archimedes," occupies, as regards both time and character, a position intermediate between these two periods. He was born at The Hague in 1629, and died there in 1695. A splendid ancestry, three years of university training at Leyden and Breda, much travel, and a rare group of associates, combined to give him an education which left little to be desired. Most of his life was spent in Holland, but for the fifteen years following 1666 he lived and worked in Paris, where he was the guest of Louis XIV. and the then recently founded French Academy of Sciences. This was for him a happy period of great activity, and it was only in anticipation of the revocation of the Edict of Nantes, in 1685, that he returned to his own country, where in private retirement and study he spent most of his remaining years.

His intellectual achievements fall into three not very distinct departments of science—namely, mathematics, physics, and physical astronomy. In mathematics, his chief accomplishments refer to—

(*a*) The quadrature of conics.

(*b*) The theory of probabilities.

(*c*) A discussion of the evolutes and involutes of curves and the introduction of the idea of the envelope of a moving straight line.

In physics he gave—

(*a*) A general solution of the problem of the Compound Pendulum, and in the demonstration enunciated the very general principle that in any mechanical system acting under gravity the centre of gravity can never rise to a point higher than that from which it fell—a principle which we now recognize as a special case of the law that the potential energy of any mechanical system tends to a minimum.

(*b*) The invention of the pendulum clock and its application to the measurement of gravity at various points on the earth's surface.

(*c*) An accurate description of the behavior of bodies in collision.

(*d*) The laws governing centrifugal forces.

(*e*) The undulatory theory of light and its application to the explanation of reflection, ordinary refraction, and double refraction.

Among his contributions to physical astronomy are—

(*a*) The construction of the first powerful telescope of the refracting kind.

(*b*) The discovery of the rings of Saturn and its sixth satellite.

(*c*) Improvements in the methods of grinding lenses and the addition of a tube to the object-glass and another to the eye-piece of the aërial telescope.

All his mechanical inventions are characterized by practicability, and all his intellectual work by clearness and elegance.

Those who wish a more detailed account of his activity will find it in the superb edition of his works* recently published by the Société Hollandaise des Sciences, while that delightful sketch of his life and work given by Dr. Bosscha† should be read by every one.

* *Œuvres Complètes de Christiaan Huygens* (La Haye: Martinus Nijhoff, 1888 to 19—).

† Bosscha: *Christiaan Huygens*, Rede am 200sten Gedächtnistage seines Lebensende. Übersetzt von Engelmann. (Engelmann: Leipzig, 1895), pp. 77.

ON THE THEORY OF LIGHT AND COLORS

From the *Philosophical Transactions* for 1802, p. 12.

AN ACCOUNT OF SOME CASES OF THE PRODUCTION OF COLORS NOT HITHERTO DESCRIBED

From the *Philosophical Transactions* for 1802, p. 387.

EXPERIMENTS AND CALCULATIONS RELATIVE TO PHYSICAL OPTICS

From the *Philosophical Transactions* for 1804.

BY

THOMAS YOUNG.

These three papers are reprinted in Young's *Miscellaneous Works*, vol. i., and also in his *Lectures on Natural Philosophy and Mechanical Arts*.

CONTENTS

ON THE THEORY OF LIGHT AND COLORS *

A BAKERIAN LECTURE

Read November 12, 1801.

ALTHOUGH the invention of plausible hypotheses, independent of any connection with experimental observations, can be of very little use in the promotion of natural knowledge, yet the discovery of simple and uniform principles, by which a great number of apparently heterogeneous phenomena are reduced to coherent and universal laws, must ever be allowed to be of considerable importance towards the improvement of the human intellect.

The object of the present dissertation is not so much to propose any opinions which are absolutely new, as to refer some theories, which have been already advanced, to their original inventors, to support them by additional evidence, and to apply them to a great number of diversified facts, which have hitherto been buried in obscurity. Nor is it absolutely necessary in this instance to produce a single new experiment; for of experiments there is already an ample store, which are so much the more unexceptionable as they must have been conducted without the least partiality for the system by which they will be explained; yet some facts, hitherto unobserved, will be brought forward, in order to show the perfect agreement of that system with the multifarious phenomena of nature.

The optical observations of Newton are yet unrivalled; and, excepting some casual inaccuracies, they only rise in our estimation as we compare them with later attempts to improve on

* From the *Philosophical Transactions* for 1802, p. 12

them. A further consideration of the colors of thin plates, as they are described in the second book of Newton's *Optics*, has converted that prepossession which I before entertained for the undulatory system of light into a very strong conviction of its truth and sufficiency, a conviction which has been since most strikingly confirmed by an analysis of the colors of striated substances. The phenomena of thin plates are indeed so singular that their general complexion is not without great difficulty reconcilable to any theory, however complicated, that has hitherto been applied to them; and some of the principal circumstances have never been explained by the most gratuitous assumptions; but it will appear that the minutest particulars of these phenomena are not only perfectly consistent with the theory which will now be detailed, but that they are all the necessary consequences of that theory, without any auxiliary suppositions; and this by inferences so simple that they become particular corollaries, which scarcely require a distinct enumeration.

A more extensive examination of Newton's various writings has shown me that he was in reality the first that suggested such a theory as I shall endeavor to maintain; that his own opinions varied less from this theory than is now almost universally supposed; and that a variety of arguments have been advanced, as if to confute him, which may be found nearly in a similar form in his own works; and this by no less a mathematician than Leonard Euler, whose system of light, as far as it is worthy of notice, either was, or might have been, wholly borrowed from Newton, Hooke, Huygens, and Malebranche.

Those who are attached, as they may be with the greatest justice, to every doctrine which is stamped with the Newtonian approbation, will probably be disposed to bestow on these considerations so much the more of their attention, as they appear to coincide more nearly with Newton's own opinions. For this reason, after having briefly stated each particular position of my theory, I shall collect, from Newton's various writings, such passages as seem to be the most favorable to its admission; and although I shall quote some papers which may be thought to have been partly retracted at the publication of the *Optics*, yet I shall borrow nothing from them that can be supposed to militate against his maturer judgment.

THE WAVE-THEORY OF LIGHT

Hypothesis I

A luminiferous ether pervades the universe, rare and elastic in a high degree.

PASSAGES FROM NEWTON

"The hypothesis certainly has a much greater affinity with his own," that is, Dr. Hooke's, "hypothesis than he seems to be aware of; the vibrations of the ether being as useful and necessary in this as in his."—*Phil. Trans.*, vol. vii., p. 5087. Abr., vol. i., p. 145. Nov., 1672.

"To proceed to the hypothesis: first, it is to be supposed therein that there is an ethereal medium, much of the same constitution with air, but far rarer, subtler, and more strongly elastic. It is not to be supposed that this medium is one uniform matter, but compounded, partly of the main phlegmatic body of ether, partly of other various ethereal spirits, much after the manner that air is compounded of the phlegmatic body of air, intermixed with various vapors and exhalations: for the electric and magnetic effluvia and gravitating principle seem to argue such variety."—Birch, *Hist. of R. S.*, vol. iii., p. 249, Dec., 1675.

"Is not the heat (of the warm room) conveyed through the vacuum by the vibrations of a much subtler medium than air? And is not this medium the same with that medium by which light is refracted and reflected, and by whose vibrations light communicates heat to bodies, and is put into fits of easy reflection and easy transmission? And do not the vibrations of this medium in hot bodies contribute to the intenseness and duration of their heat? And do not hot bodies communicate their heat to contiguous cold ones by the vibrations of this medium propagated from them into the cold ones? And is not this medium exceedingly more rare and subtle than the air, and exceedingly more elastic and active? And doth it not readily pervade all bodies? And is it not, by its elastic force, expanded through all the heavens? May not planets and comets, and all the gross bodies, perform their motions in this ethereal medium? And may not its resistance be so small as to be inconsiderable? For instance, if this ether (for so I will call it) should be supposed 700,000 times more elastic than our air, and above 700,000 times more rare, its resistance would be about 600,000,000 times less than that of water. And so small a resistance would scarce make any sensible alteration in the motions of the planets in ten thousand years. If any one would ask how a medium can be so rare, let him tell me how an electric body can by friction emit an exhalation so rare and subtle, and yet so potent? And how the effluvia of a magnet can pass through a plate of glass without resistance, and yet turn a magnetic needle beyond the glass?"—*Optics*, Qu. 18, 22.

Hypothesis II

Undulations are excited in this ether whenever a body becomes luminous.

Scholium. I use the word undulation in preference to vibration, because vibration is generally understood as implying a motion which is continued alternately backward and forward by a combination of the momentum of the body with an accelerating force, and which is naturally more or less permanent; but an undulation is supposed to consist in vibratory motion transmitted successively through different parts of a medium without any tendency in each particle to continue its motion, except in consequence of the transmission of succeeding undulations from a distinct vibrating body; as in the air the vibrations of a chord produce the undulations constituting sound.

PASSAGES FROM NEWTON

"Were I to assume an hypothesis, it should be this, if propounded more generally, so as not to determine what light is further than that it is something or other capable of exciting vibrations in the ether; for thus it will become so general and comprehensive of other hypotheses as to leave little room for new ones to be invented."—Birch, *Hist. of R. S.*, vol. iii., p. 249, Dec., 1675.

"In the second place, it is to be supposed that the ether is a vibrating medium like air, only the vibrations far more swift and minute; those of air, made by a man's ordinary voice, succeeding one another at more than half a foot or a foot distance, but those of ether at a less distance than the hundred-thousandth part of an inch. And as in air the vibrations are some larger than others, but yet all equally swift (for in a ring of bells the sound of every tone is heard at two or three miles distance in the same order that the bells are struck), so, I suppose, the ethereal vibrations differ in bigness, but not in swiftness. Now, these vibrations, besides their use in reflection and refraction, may be supposed the chief means by which the parts of fermenting or putrefying substances, fluid liquors, or melted, burning, or other hot bodies, continue in motion."—BIRCH, *Hist. of R. S.*, vol. iii., p. 251, Dec., 1675.

"When a ray of light falls upon the surface of any pellucid body, and is there refracted or reflected, may not waves of vibrations, or tremors, be thereby excited in the refracting or reflecting medium? And are not these vibrations propagated from the point of incidence to great distances? And do they not overtake the rays of light, and by overtaking them successively, do not they put them into the fits of easy reflection and easy transmission described above?"—*Optics*, Qu. 17.

"Light is in fits of easy reflection and easy transmission before its incidence on transparent bodies. And probably it is put into such fits at its first emission from luminous bodies, and continues in them during all its progress."—*Optics*, Book ii., part iii., prop. 13.

HYPOTHESIS III

The sensation of different colors depends on the different frequency of vibrations excited by light in the retina.

PASSAGES FROM NEWTON

"The objector's hypothesis, as to the fundamental part of it, is not against me. That fundamental supposition is, that the parts of bodies, when briskly agitated, do excite vibrations in the ether, which are propagated every way from those bodies in straight lines, and cause a sensation of light by beating and dashing against the bottom of the eye, something after the manner that vibrations in the air cause a sensation of sound by beating against the organs of hearing. Now, the most free and natural application of this hypothesis to the solution of phenomena I take to be this—that the agitated parts of bodies, according to their several sizes, figures, and motions, do excite vibrations in the ether of various depths or bignesses, which, being promiscuously propagated through that medium to our eyes, effect in us a sensation of light of a white color; but if by any means those of unequal bignesses be separated from one another, the largest beget a sensation of a red color; the least, or shortest, of a deep violet, and the intermediate ones of intermediate colors; much after the manner that bodies, according to their several sizes, shapes, and motions, excite vibrations in the air of various bignesses, which, according to those bignesses, make several tones in sound: that the largest vibrations are best able to overcome the resistance of a refracting superficies, and so break through it with least refraction; whence the vibrations of several bignesses—that is, the rays of several colors, which are blended together in light—must be parted from one another by refraction, and so cause the phenomena of prisms and other refracting substances; and that it depends on the thickness of a thin transparent plate or bubble whether a vibration shall be reflected at its further superficies or transmitted; so that, according to the number of vibrations interceding the two superficies, they may be reflected or transmitted for many successive thicknesses. And since the vibrations which make blue and violet are supposed shorter than those which make red and yellow, they must be reflected at a less thickness of the plate, which is sufficient to explicate all the ordinary phenomena of those plates or bubbles, and also of all natural bodies, whose parts are like so many fragments of such plates. These seem to be the most plain, genuine, and necessary conditions of this hypothesis; and they agree so justly with my theory that if the animadversor think fit to apply them, he need not, on that account, apprehend a divorce from it; but yet, how he will defend it from other difficulties I know not."—*Phil. Trans.*, vol. vii., p. 5088. Abr., vol. i., p. 145. Nov., 1672.

"To explain colors, I suppose that as bodies of various sizes, densities, or sensations do by percussion or other action excite sounds of various tones, and consequently vibrations in the air of different bigness, so the rays of light, by impinging on the stiff refracting superficies, excite vibrations in the ether of various bigness, the biggest, strongest, or most potent rays, the largest vibrations; and others shorter, according to their bigness, strength, or power: and therefore the ends of the capillamenta of the optic nerve, which pave or face the retina, being such refracting superficies, when the rays impinge upon them, they must there excite these vibrations, which vibrations (like those of sound in a trunk or trumpet) will run along the aqueous pores or crystalline pith of the capillamenta, through the optic nerves, into the sensorium; and there, I suppose, affect the sense with various colors, according to their bigness and mixture; the biggest with the strongest colors, reds and yellows; the least with the weakest—blues and violets; the middle with green, and a confusion of all with white—much after the manner that, in the sense of hearing, nature makes use of aërial vibrations of several bignesses to generate sounds of divers tones, for the analogy of nature is to be observed."—BIRCH, *Hist. of R. S.*, vol. iii., p. 262, Dec., 1675.

"Considering the lastingness of the motions excited in the bottom of the eye by light, are they not of a vibrating nature? Do not the most refrangible rays excite the shortest vibrations, the least refrangible the largest? May not the harmony and discord of colors arise from the proportions of the vibrations propagated through the fibres of the optic nerve into the brain, as the harmony and discord of sounds arise from the proportions of the vibrations of the air?"—*Optics*, Qu. 16, 13, 14.

[*Scholium omitted.*]

HYPOTHESIS IV

All material bodies have an attraction for the ethereal medium, by means of which it is accumulated within their substance, and for a small distance around them, in a state of greater density but not of greater elasticity.

It has been shown that the three former hypotheses, which may be called essential, are literally parts of the more complicated Newtonian system. This fourth hypothesis differs perhaps, in some degree from any that have been proposed by former authors, and is diametrically opposite to that of Newton; but both being in themselves equally probable, the opposition is merely accidental, and it is only to be inquired which is the best capable of explaining the phenomena. Other suppositions might perhaps be substituted for this, and therefore I do not consider it as fundamental, yet it appears to be the simplest and best of any that have occurred to me.

Proposition I

All impulses are propagated in a homogeneous elastic medium with an equable velocity.

Every experiment relative to sound coincides with the observation already quoted from Newton, that all undulations are propagated through the air with equal velocity; and this is further confirmed by calculations. (Lagrange, *Misc. Taur.*, vol. i., p. 91. Also, much more concisely, in my syllabus of a course of lectures on Natural and Experimental Philosophy, about to be published. Art. 289.) If the impulse be so great as materially to disturb the density of the medium, it will be no longer homogeneous; but, as far as concerns our senses, the quantity of motion may be considered as infinitely small. It is surprising that Euler, although aware of the matter of fact, should still have maintained that the more frequent undulations are more rapidly propagated. (*Theor. mus.* and *Conject. phys.*) It is possible that the actual velocity of the particles of the luminiferous ether may bear a much less proportion to the velocity of the undulations than in sound, for light may be excited by the motion of a body moving at the rate of only one mile in the time that light moves a hundred millions.

Scholium 1. It has been demonstrated that in different mediums the velocity varies in the subduplicate ratio of the force directly and of the density inversely. (*Misc. Taur.*, vol. i., p. 91. Young's *Syllabus*. Art. 294.)

Scholium 2. It is obvious, from the phenomena of elastic bodies and of sounds, that the undulations may cross each other without interruption; but there is no necessity that the various colors of white light should intermix their undulations, for, supposing the vibrations of the retina to continue but a five-hundredth of a second after their excitement, a million undulations of each of a million colors may arrive in distinct succession within this interval of time, and produce the same sensible effect as if all the colors arrived precisely at the same instant.

Proposition II

An undulation conceived to originate from the vibration of a single particle must expand through a homogeneous medium *in a spherical form, but with different quantities of motion in different parts.*

For, since every impulse, considered as positive or negative, is propagated with a constant velocity, each part of the undulation must in equal times have passed through equal distances from the vibrating-point. And, supposing the vibrating particle, in the course of its motion, to proceed forward to a small distance in a given direction, the principal strength of the undulation will naturally be straight before it; behind it the motion will be equal in a contrary direction; and at right angles to the line of vibration the undulation will be evanescent.

Now, in order that such an undulation may continue its progress to any considerable distance, there must be in each part of it a tendency to preserve its own motion in a right line from the centre; for if the excess of force at any part were communicated to the neighboring particles, there can be no reason why it should not very soon be equalized throughout, or, in other words, become wholly extinct, since the motions in contrary directions would naturally destroy each other. The origin of sound from the vibration of a chord is evidently of this nature; on the contrary, in a circular wave of water every part is at the same instant either elevated or depressed. It may be difficult to show mathematically the mode in which this inequality of force is preserved, but the inference from the matter of fact appears to be unavoidable; and while the science of hydrodynamics is so imperfect that we cannot even solve the simple problem of the time required to empty a vessel by a given aperture, it cannot be expected that we should be able to account perfectly for so complicated a series of phenomena as those of elastic fluids. The theory of Huygens, indeed, explains the circumstance in a manner tolerably satisfactory. He supposes every particle of the medium to propagate a distinct undulation in all directions, and that the general effect is only perceptible where a portion of each undulation conspires in direction at the same instant; and it is easy to show that such a general undulation would in all cases proceed rectilinearly, with proportionate force; but, upon this supposition, it seems to follow that a greater quantity of force must be lost by the divergence of the partial undulations than appears to be consistent with the propagation of the effect to any considerable distance; yet it is obvious that some such limitation of the motion must naturally be expected to take place, for if the intensity of the motion of any particular part, instead of continuing to be propagated straight forward, were supposed to affect the intensity of a neighboring part of the undulation, an impulse must then have travelled from an internal to an external circle in an oblique direction, in the same time as in the direction of the radius, and consequently with a greater velocity, against the first proposition. In the case of water the velocity is by no means so rigidly limited as in that of an elastic medium. Yet it is not necessary to suppose, nor is it indeed probable, that there is absolutely not the least lateral communication of the force of the undulation, but that, in highly elastic mediums, this communication is almost insensible. In the air, if a chord be perfectly insulated so as to propagate exactly such vibrations as have been described, they will, in fact, be much less forcible than if the chord be placed in the neighborhood of a sounding-board, and probably in some measure because of this lateral communication of motions of an opposite tendency. And the different intensity of different parts of the same circular undulation may be observed by holding a common tuning-fork at arm's-length, while sounding, and turning it, from a plane directed to the ear, into a position perpendicular to that plane.

Proposition III

A portion of a spherical undulation, admitted through an aperture into a quiescent medium, will proceed to be further propagated rectilinearly in concentric superficies, terminated laterally by weak and irregular portions of newly diverging undulations.

At the instant of admission the circumference of each of the undulations may be supposed to generate a partial undulation, filling up the nascent angle between the radii and the surface terminating the medium; but no sensible addition will be made to its strength by a divergence of motion from any other parts of the undulation, for want of a coincidence in time, as has already been explained with respect to the various force of a spherical undulation. If, indeed, the aperture bear but a small proportion to the breadth of an undulation, the newly generated undulation may nearly absorb the whole force of the portion admitted; and this is the case considered by Newton in the *Principia.* But no experiment can be made under these circumstances with light, on account of the minuteness of its undulations and the interference of inflection; and yet some faint radiations do actually diverge beyond any probable limits of inflection, rendering the margin of the aperture distinctly visible in all directions. These are attributed by Newton to some unknown cause, distinct from inflection (*Optics,* Book iii., obs. 5), and they fully answer the description of this proposition.

Let the concentric lines in Fig. 13 represent the contemporaneous situation of similar parts of a number of successive undulations diverging from the point A; they will also represent the successive situations of each individual undulation: let the force of each undulation be represented by the breadth of the line, and let the cone of light ABC be admitted through the aperture BC; then the principal undulations will proceed in a rectilinear direction towards GH, and the faint radiations on each side will diverge from B and C as centres, without receiving any additional force from any intermediate point D of the undulation, on account of the inequality of the lines DE and DF. But if we allow some little lateral divergence from the extremities of the undulations, it must diminish their force, without adding materially to that of the dissipated light; and their termination, instead of the right line BG, will assume the form CH, since the loss of force must be more considerable near to C than at greater distances. This line corresponds with the boundary of the shadow in Newton's first observation, Fig. 13; and it is much more probable that such a dissipation of light was the cause of the increase of the shadow in that observation than that it was owing to the action of the inflecting atmosphere, which must have extended a thirtieth of an inch each way in order to produce it; especially when it is considered that

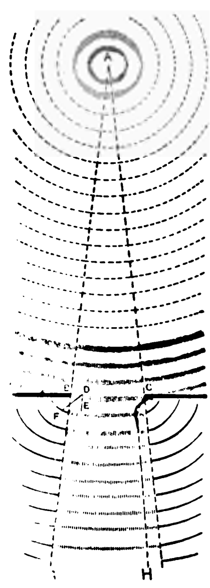


Fig. 13

the shadow was not diminished by surrounding the hair with a denser medium than air, which must in all probability have weakened and contracted its inflecting atmosphere. In other circumstances the lateral divergence might appear to increase, instead of diminishing, the breadth of the beam.

As the subject of this proposition has always been esteemed the most difficult part of the undulatory system, it will be proper to examine here the objections which Newton has grounded upon it.

"To me the fundamental supposition itself seems impossible—namely, that the waves or vibrations of any fluid can, like the rays of light, be propagated in straight lines, without a continual and very extravagant spreading and bending every way into the quiescent medium, where they are terminated by it. I mistake if there be not both experiment and demonstration to the contrary."—*Phil. Trans.*, vol. vii., p. 5089. Abr., vol. i., p. 146. Nov. 1672.

"Motus omnis per fluidum propagatus divergit a recto tramite in spatia immota."

"Quoniam medium ibi," in the middle of an undulation admitted, "densius est, quam in spatiis hinc inde, dilatabit sese tam versus spatia utrinque sita, quam versus pulsum rariora intervalla; eoque pacto—pulsus eadem *fere* celeritate sese in medii partes quiescentes hinc inde relaxare debent;—ideoque spatium totum occupabunt—Hoc experimur in sonis."—*Princip.*, lib. ii., prop. 42.

"Are not all hypotheses erroneous in which light is supposed to consist in pression or motion propagated through a fluid medium? If it consisted in pression or motion, propagated either in an instant, or in time, it would bend into the shadow. For pression or motion cannot be propagated in a fluid in right lines beyond an obstacle which stops part of the motion, but will bend and spread every way into the quiescent medium which lies beyond the obstacle. The waves on the surface of stagnating water passing by the sides of a broad obstacle which stops part of them, bend afterwards, and dilate themselves gradually into the quiet water behind the obstacle. The waves, pulses, or vibrations of the air, wherein sounds consist, bend manifestly, though not so much as the waves of water. For a bell or a cannon may be heard beyond a hill which intercepts the sight of the sounding body; and sounds are propagated as readily through crooked pipes as straight ones. But light is never known to follow crooked passages nor to bend into the shadow. For the fixed stars, by the interposition of any of the planets, cease to be seen. And so do the parts of the sun by the interposition of the moon, Mercury, or Venus. The rays which pass very near to the edges of any body are bent a little by the action of the body; but this bending is not towards but from the shadow, and is performed only in the passage of the ray by the body, and at a very small distance from it. So soon as the ray is past the body it goes right on."—*Optics*, Qu. 28.

Now the proposition quoted from the *Principia* does not directly contradict this proposition; for it does not assert that such a motion must diverge equally in all directions; neither can it with truth be maintained that the parts of an elastic medium communicating any motion must propagate that motion equally in all directions. All that can be inferred by reasoning is that the marginal parts of the undulation must be somewhat weakened and that there must be a faint divergence in every direction; but whether either of these effects might be of sufficient magnitude to be sensible could not have been inferred from argument, if the affirmative had not been rendered probable by experiment.

As to the analogy with other fluids, the most natural inference from it is this: "The waves of the air, wherein sounds consist, bend manifestly, though not so much as the waves of water"; water being an inelastic and air a moderately elastic medium; but ether being most highly elastic, its waves bend very far less than those of the air, and therefore almost imperceptibly. Sounds are propagated through crooked passages, because their sides are capable of reflecting sound, just as light would be propagated through a bent tube, if perfectly polished within.

The light of a star is by far too weak to produce, by its faint divergence, any visible illumination of the margin of a planet eclipsing it; and the interception of the sun's light by the moon is as foreign to the question as the statement of inflection is inaccurate.

To the argument adduced by Huygens in favor of the rectilinear propagation of undulations Newton has made no reply; perhaps because of his own misconception of the nature of the motions of elastic mediums, as dependent on a peculiar law of vibration, which has been corrected by later mathematicians. On the whole, it is presumed that this proposition may be safely admitted as perfectly consistent with analogy and with experiment.

Proposition IV

When an undulation arrives at a surface which is the limit of mediums of different densities, a partial reflection takes place proportionate in force to the difference of the densities.

This may be illustrated, if not demonstrated, by the analogy of elastic bodies of different sizes. "If a smaller elastic body strikes against a larger one, it is well known that the smaller is reflected more or less powerfully, according to the difference of their magnitudes: thus, there is always a reflection when the rays of light pass from a rarer to a denser stratum of ether; and frequently an echo when a sound strikes against a cloud. A greater body striking a smaller one propels it, without losing all its motion: thus, the particles of a denser stratum of ether do not impart the whole of their motion to a rarer, but, in their effort to proceed, they are recalled by the attraction of the refracting substance with equal force; and thus a reflection is always secondarily produced when the rays of light pass from a denser to a rarer stratum." But it is not absolutely necessary to suppose an attraction in the latter case, since the effort to proceed would be propagated backward without it, and the undulation would be reversed, a rarefaction returning in place of a condensation; and this will perhaps be found most consistent with the phenomena.

[*Propositions V., VI., and VII. omitted.*]

Proposition VIII

When two undulations, from different origins, coincide either perfectly or very nearly in direction, their joint effect is a combination of the motions belonging to each.

Since every particle of the medium is affected by each undulation, wherever the directions coincide, the undulations can proceed no otherwise than by uniting their motions, so that the joint motion may be the sum or difference of the separate motions, accordingly as similar or dissimilar parts of the undulations are coincident.

I have, on a former occasion, insisted at large on the application of this principle to harmonics; and it will appear to be of still more extensive utility in explaining the phenomena of colors. The undulations which are now to be compared are those of equal frequency. When the two series coincide exactly in point of time, it is obvious that the united velocity of the particular motions must be greatest, and, in effect at least, double the separate velocities; and also that it must be smallest, and, if the undulations are of equal strength, totally destroyed when the time of the greatest direct motion belonging to one undulation coincides with that of the greatest retrograde motion of the other. In intermediate states the joint undulation will be of intermediate strength; but by what laws this intermediate strength must vary cannot be determined without further data. It is well known that a similar cause produces in sound that effect which is called a beat; two series of undulations of nearly equal magnitude co-operating and destroying each other alternately, as they coincide more or less perfectly in the times of performing their respective motions.

[*Proposition IX. and five corollaries to Proposition VIII. are here omitted.*]

AN ACCOUNT OF SOME CASES OF THE PRODUCTION OF COLORS NOT HITHERTO DESCRIBED*

READ JULY 1, 1802

WHATEVER opinion may be entertained of the theory of light and colors which I have lately had the honor of submitting to the Royal Society, it must at any rate be allowed that it has given birth to the discovery of a simple and general law capable of explaining a number of the phenomena of colored light, which, without this law, would remain insulated and unintelligible. The law is, that "wherever two portions of the same light arrive at the eye by different routes, either exactly or very nearly in the same direction, the light becomes most intense when the difference of the routes is any multiple of a certain length, and least intense in the intermediate state of the interfering portions; and this length is different for light of different colors."

I have already shown in detail the sufficiency of this law for explaining all the phenomena described in the second and third books of Newton's *Optics*, as well as some others not mentioned by Newton. But it is still more satisfactory to observe its conformity to other facts, which constitute new and distinct classes of phenomena, and which could scarcely have agreed so well with any anterior law, if that law had been erroneous or imaginary: these are the colors of fibres and the colors of mixed plates.

As I was observing the appearance of the fine parallel lines of light which are seen upon the margin of an object held near

* From the *Philosophical Transactions* for 1802, p. 387.

the eye, so as to intercept the greater part of the light of a distant luminous object, and which are produced by the fringes caused by the inflection of light already known, I observed that they were sometimes accompanied by colored fringes, much broader and more distinct; and I soon found that these broader fringes were occasioned by the accidental interposition of a hair. In order to make them more distinct, I employed a horse-hair, but they were then no longer visible. With a fibre of wool, on the contrary, they became very large and conspicuous; and, with a single silk-worm's thread, their magnitude was so much increased that two or three of them seemed to occupy the whole field of view. They appeared to extend on each side of the candle, in the same order as the colors of thin plates seen by transmitted light. It occurred to me that their cause must be sought in the interference of two portions of light, one reflected from the fibre, the other bending round its opposite side, and at last coinciding nearly in direction with the former portion; that, accordingly, as both portions deviated more from a rectilinear direction, the difference of the length of their paths would become gradually greater and greater, and would consequently produce the appearances of color usual in such cases; that supposing them to be inflected at right angles, the difference would amount nearly to the diameter of the fibre, and that this difference must consequently be smaller as the fibre became smaller; and, the number of fringes in a right angle becoming smaller, that their angular distances would consequently become greater, and the whole appearance would be dilated. It was easy to calculate that for the light least inflected the difference of the paths would be to the diameter of the fibre very nearly as the deviation of the ray at any point from the rectilinear direction to its distance from the fibre.

I therefore made a rectangular hole in a card, and bent its ends so as to support a hair parallel to the sides of the hole; then, upon applying the eye near the hole, the hair, of course, appeared dilated by indistinct vision into a surface, of which the breadth was determined by the distance of the hair and the magnitude of the hole, independently of the temporary aperture of the pupil. When the hair approached so near to the direction of the margin of a candle that the inflected light was sufficiently copious to produce a sensible effect, the fringes began to appear; and it was easy to estimate the proportion of their breadth to the apparent breadth of the hair across the image of which they extended. I found that six of the brightest red fringes, nearly at equal distances, occupied the whole of that image. The breadth of the aperture was $\frac{66}{1000}$, and its distance from the hair $\frac{8}{10}$ of an inch; the diameter of the hair was less than $\frac{1}{500}$ of an inch; as nearly as I could ascertain it was $\frac{1}{600}$. Hence, we have $\frac{11}{1000}$ for the deviation of the first red fringe at the distance $\frac{8}{10}$; and as $\frac{8}{10} : \frac{11}{1000} : : \frac{1}{600} : \frac{11}{480000}$, or $\frac{1}{43636}$ for the difference of the routes of the red light where it was most intense. The measure deduced from Newton's experiments is $\frac{1}{39200}$. I thought this coincidence, with only an error of one-ninth of so minute a quantity, sufficiently perfect to warrant completely the explanation of the phenomenon, and even to render a repetition of the experiment unnecessary; for there are several circumstances which make it difficult to calculate much more precisely what ought to be the result of the measurement.

When a number of fibres of the same kind—for instance, a uniform lock of wool—are held near to the eye, we see an appearance of halos surrounding a distant candle; but their brilliancy, and even their existence, depends on the uniformity of the dimensions of the fibres; and they are larger as the fibres are smaller. It is obvious that they are the immediate consequences of the coincidence of a number of fringes of the same size, which, as the fibres are arranged in all imaginable directions, must necessarily surround the luminous object at equal distances on all sides, and constitute circular fringes.

There can be little doubt that the colored atmospherical halos are of the same kind; their appearance must depend on the existence of a number of particles of water of equal dimensions, and in a proper position with respect to the luminary and to the eye. As there is no natural limit to the magnitude of the spherules of water, we may expect these halos to vary without limit in their diameters, and accordingly Mr. Jordan has observed that their dimensions are exceedingly various, and has remarked that they frequently change during the time of observation.

I first noticed the colors of mixed plates in looking at a candle through two pieces of plate-glass with a little moisture between them. I observed an appearance of fringes resembling the common colors of thin plates; and, upon looking for the fringes by reflection, I found that these new fringes were always in the same direction as the other fringes, but many times larger. By examining the glasses with a magnifier, I perceived that wherever these fringes were visible the moisture was intermixed with portions of air, producing an appearance similar to dew. I then supposed that the origin of the colors was the same as that of the colors of halos; but, on a more minute examination, I found that the magnitude of the portions of air and water was by no means uniform, and that the explanation was, therefore, inadmissible. It was, however, easy to find two portions of light sufficient for the production of these fringes; for the light transmitted through the water, moving in it with a velocity different from that of the light passing through the interstices filled only with air, the two portions would interfere with each other and produce effects of color according to the general law. The ratio of the velocities in water and in air is that of 3 to 4; the fringes ought, therefore, to appear where the thickness is six times as great as that which corresponds to the same color in the common case of thin plates; and, upon making the experiment with a plane glass and a lens slightly convex, I found the sixth dark circle actually of the same diameter as the first in the new fringes. The colors are also very easily produced when butter or tallow is substituted for water; and the rings then become smaller, on account of the greater refractive density of the oils; but when water is added, so as to fill up the interstices of the oil, the rings are very much enlarged; for here the difference only of the velocities in water and in oil is to be considered, and this is much smaller than the difference between air and water. All these circumstances are sufficient to satisfy us with respect to the truth of the explanation; and it is still more confirmed by the effect of inclining the plates to the direction of the light; for then, instead of dilating, like the colors of thin plates, these rings contract: and this is the obvious consequence of an increase of the length of the paths of light, which now traverse both mediums obliquely; and the effect is everywhere the same as that of a thicker plate.

It must, however, be observed that the colors are not produced in the whole light that is transmitted through the mediums: a small portion only of each pencil, passing through the water contiguous to the edges of the particle, is sufficiently coincident with the light transmitted by the neighboring portions of air to produce the necessary interference; and it is easy to show that, on account of the natural concavity of the surface of each portion of the fluid adhering to the two pieces of glass, a considerable portion of the light which is beginning to pass through the water will be dissipated laterally by reflection at its entrance, and that much of the light passing through the air will be scattered by refraction at the second surface. For these reasons the fringes are seen when the plates are not directly interposed between the eye and the luminous object; and on account of the absence of foreign light, even more distinctly than when they are in the same right line with that object. And if we remove the plates to a considerable distance out of this line, the rings are still visible and become larger than before; for here the actual route of the light passing through the air is longer than that of the light passing more obliquely through the water, and the difference in the times of passage is lessened. It is, however, impossible to be quite confident with respect to the causes of these minute variations, without some means of ascertaining accurately the forms of the dissipating surfaces.

In applying the general law of interference to these colors, as well as to those of thin plates already known, I must confess that it is impossible to avoid another supposition, which is a part of the undulatory theory—that is, that the velocity of light is the greater the rarer the medium; and that there is also a condition annexed to the explanation of the colors of thin plates which involves another part of the same theory—that is, that where one of the portions of light has been reflected at the surface of a rarer medium, it must be supposed to be retarded one-half of the appropriate interval—for instance, in the central black spot of a soap-bubble, where the actual lengths of the paths very nearly coincide, but the effect is the same as if one of the portions had been so retarded as to destroy the other. From considering the nature of this circumstance, I ventured to predict that if the two reflections were of the same kind, made at the surfaces of a thin plate of a density intermediate between the densities of the mediums containing it, the effect would be reversed, and the central spot, instead of black, would become white; and I have now the pleasure of stating that I have fully verified this prediction by interposing a drop of oil of sassafras between a prism of flint-glass and a lens of crown-glass; the central spot seen by reflected light was white and surrounded by a dark ring. It was, however, necessary to use some force in order to produce a contact sufficiently intimate; and the white spot differed, even at last, in the same degree from perfect whiteness as the black spot usually does from perfect blackness.

[*Three pages of speculation concerning dispersion are here omitted.*]

EXPERIMENTS AND CALCULATIONS RELATIVE TO PHYSICAL OPTICS*

A BAKERIAN LECTURE

Read November 24, 1803

I. EXPERIMENTAL DEMONSTRATION OF THE GENERAL LAW OF THE INTERFERENCE OF LIGHT.

IN making some experiments on the fringes of colors accompanying shadows, I have found so simple and so demonstrative a proof of the general law of the interference of two portions of light, which I have already endeavored to establish, that I think it right to lay before the Royal Society a short statement of the facts which appear to me so decisive. The proposition on which I mean to insist at present is simply this—that fringes of colors are produced by the interference of two portions of light; and I think it will not be denied by the most prejudiced that the assertion is proved by the experiments I am about to relate, which may be repeated with great ease whenever the sun shines, and without any other apparatus than is at hand to every one.

Experiment 1. I made a small hole in a window-shutter, and covered it with a piece of thick paper, which I perforated with a fine needle. For greater convenience of observation I placed a small looking-glass without the window-shutter, in such a position as to reflect the sun's light in a direction nearly horizontal upon the opposite wall, and to cause the cone of diverging light to pass over a table on which were several little screens of card-paper. I brought into the sunbeam a slip of

* From the *Philosophical Transactions* for 1804.

card about one-thirtieth of an inch in breadth, and observed its shadow, either on the wall or on other cards held at different distances. Besides the fringes of color on each side of the shadow, the shadow itself was divided by similar parallel fringes of smaller dimensions, differing in number according to the distance at which the shadow was observed, but leaving the middle of the shadow always white. Now these fringes were the joint effects of the portions of light passing on each side of the slip of card, and inflected, or rather diffracted, into the shadow; for a little screen being placed a few inches from the card so as to receive either edge of the shadow on its margin, all the fringes which had before been observed in the shadow on the wall immediately disappeared, although the light inflected on the other side was allowed to retain its course, and although this light must have undergone any modification that the proximity of the other edge of the slip of card might have been capable of occasioning. When the interposed screen was more remote from the narrow card, it was necessary to plunge it more deeply into the shadow, in order to extinguish the parallel lines; for here the light diffracted from the edge of the object had entered farther into the shadow in its way towards the fringes. Nor was it for want of a sufficient intensity of light that one of the two portions was incapable of producing the fringes alone; for when they were both uninterrupted, the lines appeared, even if the intensity was reduced to one-tenth or one-twentieth.

Experiment 2. The crested fringes described by the ingenious and accurate Grimaldi afford an elegant variation of the preceding experiment and an interesting example of a calculation grounded on it. When a shadow is formed by an object which has a rectangular termination besides the usual external fringes there are two or three alternations of colors, beginning from the line which bisects the angle, disposed on each side of it in curves, which are convex towards the bisecting line, and which converge in some degree towards it as they become more remote from the angular point. These fringes are also the joint effect of the light which is inflected directly towards the shadow from each of the two outlines of the object; for if a screen be placed within a few inches of the object, so as to receive only one of the edges of the shadow, the whole of the fringes disappear; if, on the contrary, the rectangular point of the screen be opposed to the point of the shadow so as barely to receive the angle of the shadow on its extremity, the fringes will remain undisturbed.

II. COMPARISON OF MEASURES DEDUCED FROM VARIOUS EXPERIMENTS.

If we now proceed to examine the dimensions of the fringes under different circumstances, we may calculate the differences of the lengths of the paths described by the portions of light which have thus been proved to be concerned in producing those fringes; and we shall find that where the lengths are equal the light always remains white; but that where either the brightest light or the light of any given color disappears and reappears a first, a second, or a third time, the differences of the lengths of the paths of the two portions are in arithmetical progression, as nearly as we can expect experiments of this kind to agree with each other. I shall compare, in this point of view, the measures deduced from several experiments of Newton and from some of my own.

In the eighth and ninth observations of the third book of Newton's *Optics* some experiments are related which, together with the third observation, will furnish us with the data necessary for the calculation. Two knives were placed, with their edges meeting at a very acute angle, in a beam of the sun's light, admitted through a small aperture, and the point of concourse of the two first dark lines bordering the shadows of the respective knives was observed at various distances. The results of six observations are expressed in the first three lines of the first table. On the supposition that the dark line is produced by the first interference of the light reflected from the edges of the knives, with the light passing in a straight line between them, we may assign, by calculating the difference of the two paths, the interval for the first disappearance of the brightest light, as it is expressed in the fourth line. The second table contains the results of a similar calculation from Newton's observations on the shadow of a hair; and the third, from some experiments of my own of the same nature; the second bright line being supposed to correspond to a double interval, the second dark line to a triple interval, and the succeeding lines to depend on a continuation of the progression. The unit of all the tables is an inch.

Table I.—*Observation* 9. N.

Distance of the knives from the aperture						101
Distance of the paper from the knives	$1\frac{1}{8}$	$3\frac{1}{8}$	$8\frac{3}{8}$	32	96	181
Distance between the edges of the knives opposite to the point of concourse	.012	.020	.034	.057	.081	.087
Interval of disappearance	.0000122	.0000155	.0000182	.0000167	.0000166	.0000166

Table II.—*Observation* 3. N.

Breadth of the hair		$\frac{1}{280}$
Distance of the hair from the aperture		144
Distances of the scale from the aperture	150	252
(Breadths of the shadow	$\frac{1}{54}$	$\frac{1}{5}$)
Breadth between the second pair of bright lines	$\frac{2}{47}$	$\frac{4}{17}$
Interval of disappearance, or half the difference of the paths	.0000151	.0000173
Breadth between the third pair of bright lines	$\frac{4}{73}$	$\frac{3}{10}$
Interval of disappearance, one-fourth of the difference	.0000130	.0000143

Table III.—*Experiment* 3.

Breadth of the object	.434
Distance of the object from the aperture	125
Distance of the wall from the aperture	250
Distance of the second pair of dark lines from each other	1.167
Interval of disappearance, one-third of the difference	.0000149

***Experiment* 4.**

Breadth of the wire	.083
Distance of the wire from the aperture	32
Distance of the wall from the aperture	250
(Breadth of the shadow, by three measurements	.815, .826, or .827; mean, .823)
Distance of the first pair of dark lines	1.165, 1.170, or 1.160; mean, 1.165
Interval of disappearance	0000194
Distance of the second pair of dark lines	1.402, 1.395, or 1.400; mean, 1.399
Interval of disappearance	.0000137
Distance of the third pair of dark lines	1.594, 1.580, or 1.585; mean, 1.586
Interval of disappearance	.0000128

It appears, from five of the six observations of the first table, in which the distance of the shadow was varied from about 3 inches to 11 feet, and the breadth of the fringes was increased in the ratio of 7 to 1, that the difference of the routes constituting the interval of disappearance varied but one-eleventh at most; and that in three out of the five it agreed with the mean, either exactly or within $\frac{1}{160}$ part. Hence we are warranted in inferring that the interval appropriate to the extinction of the brightest light is either accurately or very nearly constant.

But it may be inferred from a comparison of all the other observations that when the obliquity of the reflection is very great some circumstance takes place which causes the interval thus calculated to be somewhat greater; thus, in the eleventh line of the third table it comes out one-sixth greater than the mean of the five already mentioned. On the other hand, the mean of two of Newton's experiments and one of mine is a result about one-fourth less than the former. With respect to the nature of this circumstance I cannot at present form a decided opinion; but I conjecture that it is a deviation of some of the light concerned, from the rectilinear direction assigned to it, arising either from its natural diffraction, by which the magnitude of the shadow is also enlarged, or from some other unknown cause. If we imagined the shadow of the wire and the fringes nearest it to be so contracted that the motion of the light bounding the shadow might be rectilinear, we should thus make a sufficient compensation for this deviation; but it is difficult to point out what precise track of the light would cause it to require this correction.

The mean of the three experiments which appear to have been least affected by this unknown deviation gives .0000127 for the interval appropriate to the disappearance of the brightest light; and it may be inferred that if they had been wholly exempted from its effects the measure would have been somewhat smaller. Now the analogous interval, deduced from the experiments of Newton on this plate, is .0000112, which is about one-eighth less than the former result; and this appears to be a coincidence fully sufficient to authorize us to attribute these two classes of phenomena to the same cause. It is very easily shown, with respect to the colors of thin plates, that each kind of light disappears and reappears where the differences of the routes of two of its portions are in arithmetical progression; and we have seen that the same law may be in general inferred from the phenomena of diffracted light, even independently of the analogy.

The distribution of the colors is also so similar in both cases as to point immediately to a similarity in the causes. In the thirteenth observation of the second part of the first book Newton relates that the interval of the glasses where the rings appeared in red light was to the interval where they appeared in violet light as 14 to 9; and, in the eleventh observation of the third book, that the distances between the fringes, under the same circumstances, were the twenty-second and the twenty-seventh of an inch. Hence, deducting the breadth of the hair and taking the squares, in order to find the relation of the difference of the routes, we have the proportion of 14 to 9¼, which scarcely differs from the proportion observed in the colors of the thin plate.

We may readily determine from this general principle the form of the crested fringes of Grimaldi, already described; for it will appear that, under the circumstances of the experiment related, the points in which the differences of the lengths of the paths described by the two portions of light are equal to a constant quantity, and in which, therefore, the same kinds of light ought to appear or disappear, are always found in equilateral hyperbolas, of which the axes coincide with the outlines of the shadow, and the asymptotes nearly with the diagonal line. Such, therefore, must be the direction of the fringes; and this conclusion agrees perfectly with the observation. But it must be remarked that the parts near the outlines of the shadow are so much shaded off as to render the character of the curve somewhat less decidedly marked where it approaches to its axis. These fringes have a slight resemblance to the hyperbolic fringes observed by Newton; but the analogy is only distant.

[*III. Application to the Supernumerary Rainbows, omitted.*]

IV. ARGUMENTATIVE INFERENCE RESPECTING THE NATURE OF LIGHT.

The experiment of Grimaldi on the crested fringes within the shadow, together with several others of his observations equally important, has been left unnoticed by Newton. Those who are attached to the Newtonian theory of light, or to the hypothesis of modern opticians founded on views still less enlarged, would do well to endeavor to imagine anything like an explanation of these experiments derived from their own doctrines; and if they fail in the attempt, to refrain at least from idle declamation against a system which is founded on the accuracy of its application to all these facts, and to a thousand others of a similar nature.

From the experiments and calculation which have been premised, we may be allowed to infer that homogeneous light at certain equal distances in the direction of its motion is possessed of opposite qualities capable of neutralizing or destroying each other, and of extinguishing the light where they happen to be united; that these qualities succeed each other alternately in successive concentric superficies, at distances which are constant for the same light passing through the same medium. From the agreement of the measures, and from the similarity of the phenomena, we may conclude that these intervals are the same as are concerned in the production of the colors of thin plates; but these are shown, by the experiments of Newton, to be the smaller the denser the medium; and since it may be presumed that their number must necessarily remain unaltered in a given quantity of light, it follows, of course, that light moves more slowly in a denser than in a rarer medium; and this being granted, it must be allowed that refraction is not the effect of an attractive force directed to a denser medium. The advocates for the projectile hypothesis of light must consider which link in this chain of reasoning they may judge to be the most feeble, for hitherto I have advanced in this paper no general hypothesis whatever. But since we know that sound diverges in concentric superficies, and that musical sounds consist of opposite qualities, capable of neutralizing each other, and succeeding at certain equal intervals, which are different according to the difference of the note, we are fully authorized to conclude that there must be some strong resemblance between the nature of sound and that of light.

I have not, in the course of these investigations, found any reason to suppose the presence of such an inflecting medium in the neighborhood of dense substances as I was formerly inclined to attribute to them; and, upon considering the phenomena of the aberration of the stars, I am disposed to believe that the luminiferous ether pervades the substance of all material bodies, with little or no resistance, as freely, perhaps, as the wind passes through a grove of trees.

The observations on the effects of diffraction and interference may, perhaps, sometimes be applied to a practical purpose in making us cautious in our conclusions respecting the appearances of minute bodies viewed in a microscope. The shadow of a fibre, however opaque, placed in a pencil of light admitted through a small aperture, is always somewhat less dark in the middle of its breadth than in the parts on each side. A similar effect may also take place, in some degree, with respect to the image on the retina, and impress the sense with an idea of a transparency which has no real existence; and if a small portion of light be really transmitted through the substance, this may again be destroyed by its interference with the diffracted light, and produce an appearance of partial opacity, instead of uniform semi-transparency. Thus a central dark spot and a light spot, surrounded by a darker circle, may respectively be produced in the images of a semi-transparent and an opaque corpuscle, and impress us with an idea of a complication of structure which does not exist. In order to detect the fallacy, we make two or three fibres cross each other, and view a number of globules contiguous to each other; or we may obtain a still more effectual remedy by changing the magnifying power; and then, if the appearance remain constant in kind and in degree, we may be assured that it truly represents the nature of the substance to be examined. It is natural to inquire whether or not the figures of the globules of blood delineated by Mr. Hewson in the *Phil. Trans.*, vol. lxiii., for 1773, might not in some measure have been influenced by a deception of this kind; but, as far as I have hitherto been able to examine the globules with a lens of one-fiftieth of an inch focus, I have found them nearly such as Mr. Hewson has described them.

[*V. Remarks on the Colors of Natural Bodies, omitted.*]

VI. EXPERIMENT ON THE DARK RAYS OF RITTER

Experiment 6. The existence of solar rays accompanying light, more refrangible than the violet rays and cognizable by their chemical effects, was first ascertained by Mr. Ritter; but Dr. Wollaston made the same experiments a very short time afterwards without having been informed of what had been done on the Continent. These rays appear to extend beyond the violet rays of the prismatic spectrum, through a space nearly equal to that which is occupied by the violet. In order to complete the comparison of their properties with those of visible light, I was desirous of examining the effect of their reflection from a thin plate of air, capable of producing the well-known rings of colors. For this purpose I formed an image of the rings, by means of the solar microscope, with the apparatus which I have described in the Journals of the Royal Institution, and I threw this image on paper dipped in a solution of nitrate of silver, placed at the distance of about nine inches from the microscope. In the course of an hour portions of three dark rings were very distinctly visible, much smaller than the brightest rings of the colored image, and coinciding very nearly in their dimensions with the rings of violet light that appeared upon the interposition of violet glass. I thought the dark rings were a little smaller than the violet rings, but the difference was not sufficiently great to be accurately ascertained; it might be as much as $\frac{1}{30}$ or $\frac{1}{40}$ of the diameters, but not greater. It is the less surprising that the difference should be so small, as the dimensions of the colored rings do not by any means vary at the violet end of the spectrum so rapidly as at the red end. For performing this experiment with very great accuracy a heliostat would be necessary, since the motion of the sun causes a slight change in the place of the image; and leather impregnated with the muriate of silver would indicate the effect with greater delicacy. The experiment, however, in its present state, is sufficient to complete the analogy of the invisible with the visible rays, and to show that they are equally liable to the general law which is the principal subject of this paper. If we had thermometers sufficiently delicate, it is probable that we might acquire, by similar means, information still more interesting with respect to the rays of invisible heat discovered by Dr. Herschel; but at present there is great reason to doubt of the practicability of such an experiment.

THE WAVE-THEORY OF LIGHT

BIOGRAPHICAL SKETCH

THOMAS YOUNG was born at Milverton, England, in 1773, and died at London in 1829. His education, in respect to the amount of ground it covered, is quite as remarkable as his later scientific work. As a lad he showed marked proficiency in linguistic studies, acquired great mechanical skill, distinguished himself in drawing, music, and athletics. As a young man he pursued his university studies at London, Edinburgh, Göttingen, and Cambridge.

The following programme of his daily work at Göttingen in the autumn of 1795 characterizes at once the lad, the youth, and the mature man:

"At 8, I attend Spittler's course on the History of the Principal States of Europe, exclusive of Germany.

"At 9, Arnemann on Materia Medica.

"At 10, Richter on Acute Diseases.

"At 11, twice a week, private lessons from Blessman, the academical dancing-master.

"At 12, I dine at Ruhlander's table d'hôte.

"At 1, twice a week, lessons on the clavichord from Forkel; and twice a week at home, from Fiorillo on Drawing.

"At 2, Lichtenberg on Physics.

"At 3, I ride in the academical manége, under the instruction of Ayrer, four times a week.

"At 4, Stromeyer on Diseases.

"At 5, Blumenbach on Natural History.

"At 6, twice Blessman with other pupils, and twice Forkel."

He was born of a well-to-do Quaker family; he inherited ample money; he had all that travel, leisure, and good society could do for a man. Only in one particular does his education appear to have been defective—*viz.*, in the absence of any training in advanced dynamics or in higher mathematical analysis.

In 1800 he completed his medical studies at Cambridge, and settled as a practising physician in London. In the year following he was appointed to the professorship of natural philosophy in the then newly founded Royal Institution, a position from which he resigned at the end of two years in order to devote himself more completely to the practice of medicine. It was during his occupancy of this chair that he published the three papers reprinted in this volume, the first of which is possibly the most important of his contributions to physics. It was during this period also that he wrote his *Lectures on Natural Philosophy,* which must always be reckoned as a potent factor in the spread of sound physical science in the nineteenth century, while its bibliography of more than four hundred quarto pages is to-day valuable as well as classic.

But nothing short of a catalogue of his papers can give one an adequate idea of the varied activity of this man during the remaining quarter-century of his life. His contributions cover fields as diverse as the physiology of the human eye, hydrodynamics, music, paleography, atmospheric refraction, theory of tides, tables of mortality, theory of structures. His explanation of color-vision as due to the presence of three sets of nerve fibres in the retina, which, when excited, give respectively sensations of red, green, and violet, has been adopted and modified by Helmholtz, and is to-day perhaps the most widely accepted of the various theories on this subject.

After all, it must be confessed, even by his most ardent admirers, that Young's style is, in general, far from clear. Whether this is in any way connected with his lack of mathematical training, or whether it is due to the fact that his own clear intuitions bridged most of the gaps in his written work, it is difficult to say; but in any event many of his papers are obscure, and few of them are read. The reader who desires a full biography will find it in Dr. Peacock's *Life of Young* (London, 1855). This biographer also edited his *Miscellaneous Works,* 3 vols. (London, 1855). All his papers, however, which are of especial interest to the student of physics are contained in the lectures on *Natural Philosophy* (London, 1807).

MEMOIR ON THE DIFFRACTION OF LIGHT

"CROWNED" BY THE FRENCH ACADEMY OF SCIENCES IN 1819

BY

A. FRESNEL

Natura simplex et fecunda.

MEMOIR ON THE ACTION OF RAYS OF POLARIZED LIGHT UPON EACH OTHER

BY

MESSRS. ARAGO AND FRESNEL

(*Annales de Chimie et de Physique.* t. x., p. 288, 1819.)

CONTENTS

FRESNEL'S PRIZE MEMOIR ON THE DIFFRACTION OF LIGHT

[*The Introduction, covering fourteen pages and describing in the most general way the defects of the emission-theory and some of the merits of the wave-theory, is omitted.*]

SECTION I

11. It might appear that on the emission-theory nothing would be simpler than the phenomena of shadows, especially when the source of light is merely a point; but, on the contrary, nothing is more complicated. If we suppose the surface of the body producing the shadow to be endowed with a repulsive property capable of changing the direction of rays of light passing very near it, we should then expect only to see the shadows increase in size and, towards their edges, to blend a little with the illuminated area; while, as a matter of fact, they are bordered with three very distinct colored fringes when one employs white light, and with a still greater number of bright and dark bands when one uses light which is practically homogeneous. These fringes we shall call *exterior*, and those which are observed in the midst of very narrow shadows we shall call *interior* fringes.

If one adopts the Newtonian theory, he is tempted at first to explain the exterior fringes as produced by a force which is alternately attractive and repulsive, and which has its source in the surface of the body producing the shadows. I shall now consider the consequences of this theory and show that its results are not justified by experiment; but, first of all, I must explain the experimental method which I have employed.

12. We know that the effect of a magnifying-glass placed in front of the eye is to reproduce accurately upon the retina any object or image which is located at its conjugate focus; at least

this is the case whenever all the rays which go to make up the image are incident upon the surface of the glass. In place, then, of projecting the fringes upon a white card or a ground glass, one may observe them directly with a magnifying-glass, and he then sees them as they are at its focus. One has then only to look towards the luminous point and place the glass between his eye and the opaque body in such a way that the point where the refracted rays cross each other falls in the middle of the pupil; this position is recognized by the fact that the entire surface of the magnifying-glass appears to be filled with light. This method is much preferable to the other two in that it enables us to study conveniently phenomena of diffraction in a weak light, and has, at the same time, the further advantage of allowing us to follow the exterior fringes right up to their source. Using a lens of 2 mm. focus and light which is practically homogeneous, I have been able to follow these fringes very close to their origin and yet observe the dark band of the fifth order. The interval which separates this band from the edge of the shadow I have measured on the micrometer and find it to be less than 0.015 mm., while the first three fringes are comprised within a space not exceed- 0.01 mm.; by using a lens of shorter focus, one would doubtless still further diminish this distance. We may thus regard the dark and bright bands as beginning at the very edge of the opaque body, so long as we do not push the accuracy of our measures beyond the hundredth part of a millimeter—an accuracy which proves to be sufficient, and which it is difficult to exceed except when the fringes are somewhat larger, as is the case with those most frequently observed.

13. This point established, suppose that we measure the exterior fringes at any given distance from the [*opaque*] screen and then allow the luminous point to approach; the fringes are observed to grow much larger. Meanwhile the angle which the incident ray passing through the origin of the fringes makes with the tangent drawn from the luminous point to the edge of the screen will be almost zero. And since these fringes take their rise at a distance less than 0.01 mm. from the edge, the variation of this angle would not be able to sensibly affect the size of the fringes. To explain this enlargement, we must therefore assume that the repulsive force increases in proportion as the opaque body approaches the luminous point. But this is

impossible, for the intensity of this force can evidently depend only upon the distance at which the light corpuscle passes the opaque body, upon the size and form of the surface of this body, upon its density, mass, or nature; and by hypothesis these all remain constant.

But even if we suppose the origin of the dark and bright bands to lie at a greater distance from the edge of the screen, a supposition which would explain the fact that they grow larger in proportion as one approaches the luminous point, it is still impossible to make the results of experiment agree with the formula deduced from the [*Newtonian*] hypothesis which we are here discussing.

14. The following table gives the distance between the darkest point in the dark band of the fourth order and the edge of the geometrical shadow* for different distances of the opaque body from the luminous point. These measures have been taken with a micrometer eye-piece which carries in its focal plane a silk fibre, the whole being moved by a micrometer screw. By the aid of a head divided into one hundred parts, passing an index, fixed with reference to the screw, one is able to read the displacement of the silk thread to within about 0.01 mm.

No. of Observation	Distance of luminous point from opaque screen	Distance of opaque body from micrometer	Distance from edge of geometrical shadow to the middle of the dark band of the fourth order
	m.	m.	mm.
1	0.100	0.7985	5.96
2	0.510	1.005	3.84
3	1.011	0.996	3.12
4	2.008	0.999	2.71
5	3.018	1.003	2.56
6	4.507	1.018	2.49
7	6.007	0.999	2.40

These experiments were made with practically homogeneous red light, which was obtained by means of a colored glass trans-

* I define *geometrical shadow* as the space included between the straight lines drawn through the luminous point and tangent to the edges of the screen; this is the shadow which the light would project if it were not diffracted.

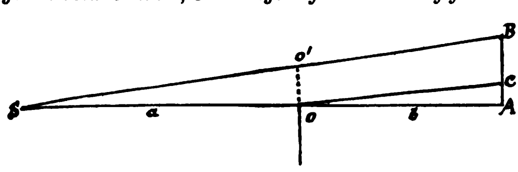
mitting only the red rays and a small portion of the orange rays. One might obtain more homogeneous light by use of a prism, but he would not be so certain as to its identity in the various observations—a condition which it is very necessary to satisfy.

15. Let us represent by a and b the respective distances of the opaque body from the luminous point and from the micrometer; let d be the distance from the edge of the body to the origin of the dark band of the fourth order, and r the tangent of the small angle of inflection resulting from the action of the repulsive forces. We then have the following expression* for the distance between the edge of the geometrical shadow and the darkest point in the dark band:

$$br + \frac{d\,(a+b)}{a}.$$

Now since r and d remain constant whatever be the distances of the luminous point from the opaque body and from the micrometer respectively, two observations suffice to determine their value. Combining the first and the last observations, we find $d = 0.5019$ mm. and $r = 1.8164$. We are thus compelled to suppose that at its origin the dark band of the fourth order is distant one-half a millimeter from the edge of the opaque body. If, now, we substitute these values in the formula and apply it to the intermediate observations, we obtain the following values, several of which evidently differ widely from the results of experiment.

* [*In diagram* S *is luminous point,* O *is edge of opaque body,* A *is edge of geometrical shadow,* O′ *is origin of dark band of fourth (or any) order. Hence*



AB *is* $\frac{d\,(a+b)}{a}$*; and the centre of the dark band is a distance* br *farther, where* AC $= br$. *The unit which Fresnel here employs for* r *is evidently one hundred times smaller than that in which we ordinarily express natural tangents.*]

No. of Observation	Distance of luminous point from opaque body	Distance of opaque body from micrometer	Distance between the edge of geometrical shadow and darkest point of fourth band		Differences
			Observed	Computed from formula $br+\frac{d\,(a+b)}{a}$	
	m.	m.	m.	mm.	mm.
1	0.1000	0.7985	5.96	——	——
2	0.510	1.005	3.84	3.32	−0.52
3	1.011	0.996	3.12	2 81	−0.31
4	2.008	0.999	2.71	2.57	−0.14
5	3.018	1.003	2.56	2.49	−0.07
6	4.507	1.018	2.49	2.46	−0.03
7	6.007	0.999	2.40	——	——

16. In attributing the production of fringes to the alternate expansion and contraction of rays which pass very near the opaque body, we are led to still another inference which is contradicted by experiment—*viz.*, that the centres of the dark and bright bands ought to lie along straight lines which would be the axes of the expanded or condensed pencils of rays. But experiment shows that in the case of exterior fringes their trajectories are hyperbolas, of which the curvature is quite sensible whenever the body which produces the shadow is sufficiently distant from the luminous point.

The screen being placed at a distance of 3.018 m. from the luminous point, I measured in succession the deviation of the darkest point of the dark band of the third order, first at 0.0017 m. from the screen, then at 1.003 m., and lastly at 3.995 m. from the screen; and I found for its distance from the edge of the geometrical shadow first 0.08 mm., secondly 2.20 mm., and thirdly 5.83 mm. If, now, we join the two extreme points by a straight line, we find for the ordinate corresponding to the intermediate point 1.52 mm. in place of 2.20 mm., the difference being 0.68 mm.—that is to say, about one and one-half times the interval between the middle of the third and the middle of the second bands. For this interval at a distance of 1.003 m. from the opaque body was only 0.42 mm., from which it is evident that the difference of 0.68 mm. cannot be attributed to an error resulting from lack of definition in the fringes observed. Nor is one able to explain this discrepancy by supposing an error in the observation made at 3.995 m. from the opaque body. From the fact that the fringes are larger, the measures should be less accurate; but in repeating them several times I find variations which at most amount to three or four hundredths of a millimeter. Even supposing that there were an error of one-half a millimeter in this measure, it would produce only a difference of 0.13 mm. at a distance of 1.003 m.; so that experiment shows conclusively that the exterior fringes lie on curved lines with their convex side outwards.

The following table gives these trajectories, referred to their chords, for different series of observations, in each of which the distance of the opaque body from the luminous point remains constant. In the fourth series I suppose first that the chord joins the two extreme readings, and next I suppose it to be drawn from the edge of the opaque body itself where the deviation of the fringes from their origin is, as we have already seen, very small. In the other series the chord joins the edge of the opaque body and the point most distant from it.

Distance from luminous point to opaque screen, or the value of *a*	Distance from opaque body to micrometer, or the value of *b*	Ordinates of Trajectories of dark bands referred to their chords				
		1st order	2d order	3d order	4th order	5th order
		1st Series				
	0	0	0	0	0	0
m.	m.	mm.	mm.	mm.	mm.	mm.
0.510	0.110	0.19	0.29	0.35	0.40	0.44
	0.501	0.14	0.21	0.25	0.30	0.34
	1.005	0	0	0	0	0
		2d Series				
	0	0	0	0	0	0
m.	m.	mm.	mm.	mm.	mm.	mm.
1.011	0.116	0.23	0.35	0.42	0.49	0.55
	0.502	0.27	0.40	0.51	0.57	0.63
	0.996	0.21	0.30	0.38	0.42	0.49
	2.010	0	0	0	0	0
		3d Series				
	0	0	0	0	0	0
m.	m.	mm.	mm.	mm.	mm.	mm.
2.008	0.118	0.26	0.38	0.47	0.54	0.60
	0.999	0.34	0.48	0.60	0.68	0.76
	2.998	0	0	0	0	0

4th Series referred to the chord joining the extreme readings

m. 3.018	m. 0.0017	0	0	0	0	0
	0.253	mm. 0.30	mm. 0.45	mm. 0 56	—	—
	0.500	0.38	0.53	0.65	—	—
	1.003	0.38	0.56	0.68	—	—
	1.998	0.31	0.45	0.54	—	—
	3.002	0.17	0.23	0.28	—	—
	3.995	0	0	0	0	0

4th Series referred to the chord drawn from the edge of the opaque body

m. 3.018	0	0	0	0	0	0
	m. 0.0017	mm. 0.04	mm. 0.06	mm. 0.08	—	—
	0.253	0.34	0.50	0.63	mm. 0.73	mm. 0.83
	0.500	0.41	0.58	0 72	0.85	0.94
	1.003	0.41	0.60	0.74	0.87	0.97
	1.998	0.32	0.48	0 57	0 67	0 75
	3.002	0.18	0.25	0 30	0 38	0.39
	3 995	0	0	0	0	0

5th Series

m. 4.507	0	0	0	0	0	0
	m. 0.131	mm. 0.27	mm. 0.40	mm. 0.50	mm. 0.58	mm. 0.66
	1.018	0 32	0.48	0.59	0 71	0.81
	2.506	0	0	0	0	0

6th Series

m. 6 007	0	0	0	0	0	0
	m. 0.117	mm. 0 23	mm. 0 33	mm. 0.42	mm. 0.49	mm. 0.53
	0.999	0	0	0	0	0

It is thus evident that the hypothesis of contraction and expansion produced by the action of the body upon rays of light is insufficient to explain the phenomena of diffraction. Introducing the principle of interference, however, we are able to predict not only the variation in size of the exterior fringes when the screen is made to approach or recede from the luminous point, but also the curved path of the bright and dark bands. The law of interference, or the mutual influence of rays of light, is an immediate consequence of the wave-theory; not only so, but it is proved or confirmed by so many different experiments that it is really one of the best-established principles of optics.

17. Grimaldi was the first to observe the effect which rays

of light produce upon one another. Recently the distinguished Dr. Thomas Young has shown by a simple and ingenious experiment that the interior fringes are produced by the meeting of rays inflected at each side of the opaque body. This he proved by using a screen to intercept one of the two pencils of light; and in this way he was able to make the interior fringes completely vanish, whatever might be the form, mass, or nature of the screen, and whether he intercepted the luminous pencil before or after its passage into the shadow.

18. Brighter and sharper fringes may be produced by cutting two parallel slits close together in a piece of cardboard or a sheet of metal, and placing the screen thus prepared in front of the luminous point. We may then observe, by use of a magnifying-glass between the opaque body and the eye, that the shadow is filled with a large number of very sharp-colored fringes so long as the light shines through both openings at the same time, but these disappear whenever the light is cut off from one of the slits.

19. If we allow two pencils of light, each coming from the same source and regularly reflected by two metallic mirrors, to meet under a very small angle, we obtain similar fringes, the colors of which are even purer and more brilliant than before. To obtain these bands, it is necessary to be very careful that in the region where the two mirrors come into contact, or at least throughout a portion of their line of contact, the surface of the one is not shifted sensibly past that of the other. This is necessary in order that the difference of path traversed by two reflected rays meeting in the area common to the two luminous* fields may be very small. I may remark in passing that the theory of interference alone will

* In the case of white light, or even in light as homogeneous as possible, the number of fringes which one can see is always limited, because even when the light has reached a degree of simplicity as great as possible without too far diminishing its intensity, it is still composed of rays which are heterogeneous; and since the bright and dark bands thus produced do not all have the same size, they encroach the one upon the other in proportion as their order increases, and finally they completely destroy each other; and this is why one does not see any fringes when the difference of paths becomes slightly sensible. Concerning the details of this experiment and its explanation on the principle of interference, see the article upon *Light* in the French translation of Thomson's Chemistry, already cited.

explain this experiment, and that the experiment calls for manipulation so delicate and effort so continued that it is almost impossible that one should strike upon it by accident.

If we raise one of the mirrors or intercept the light which it reflects either before or after reflection, the fringes disappear as in the preceding case. This furnishes still further evidence that the fringes are produced, not by the action of the edges of the mirrors, but by the meeting of two pencils of light. For these fringes are always at right angles to the line which joins the two images of the luminous point, whatever be its inclination with respect to these edges, at least throughout the extent of the area which is common to the two regularly reflected pencils.*

20. Since the fringes which one sees in the interior of the shadow of a very narrow body and those which one obtains by the use of two mirrors result evidently from the mutual influence of rays of light, analogy would indicate that the same thing ought to be true for the exterior fringes of the shadows of bodies illuminated by a point source. The first explanation which occurs to one is that these fringes are produced by the interference of direct rays with those which are reflected at the edge of the opaque body, while the interior fringes result from the combined action of rays inflected into the shadow from the two sides of the opaque body, these inflected rays having their origin either at the surface or at points indefinitely near it. This appears to be the opinion of Mr. Young, and it was at first my own opinion; but a closer examination of the phenomena convinced me of its falsity. Nevertheless, I propose to follow it to its logical conclusion and to state the formula which I have derived in order to facilitate comparison of this theory with that which I offer as a substitute.

Let R, Fig. 14, be the radiant point, AA′ the opaque body, and FT′ either a white screen upon which the shadow of this body falls or the focal plane of a magnifying-glass with which the

* When the fringes extend outside, all their exterior portions resulting from the meeting of rays regularly reflected by one of the mirrors and rays inflected near the edge of the other should have different directions. If one observes this phenomenon carefully, he will see that the form and position of the fringes are in each case in accord with the theory of interference.

fringes are observed. RT and RT′ are rays tangent at the edge of the opaque body, T and T′ being the limits of the geometrical shadow. Let us indicate by a the distance RB from the luminous point to the opaque body, by b the distance BC of the body from the white screen, and by c its diameter, AA′, which we shall consider very small compared with the distances a and b. This assumption is made in order that we may measure the size of the fringes either in a plane perpendicular to RT or perpendicular to the line RC, which passes through the middle of the shadow.

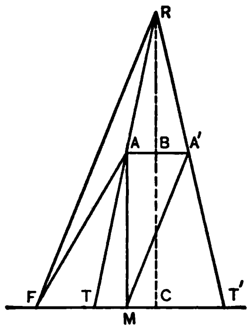


Fig. 14

With these conventions we shall consider, first, the exterior fringes. Let F be any point on the receiving screen outside the shadow. The difference of path traversed by the direct ray, and by the ray reflected at the edge of the opaque body, and meeting the direct ray at this point, is RA+AF−RF. Let us represent FT by x, and express in series the values of RF, AR, and AF. Then, if we neglect all terms involving any power of x or of c higher than the second, since they are very small compared with distances a and b, the terms which contain c will disappear and we shall have for the difference of path traversed

$$d=\frac{a}{2b\,(a+b)}x^2\,;$$

whence follows

$$x=\sqrt{\frac{2db(a+b)}{a}}.$$

21. If we call λ the length of a light-wave, that is to say, the distance between two points in the ether where vibrations of the same kind are occurring at the same time and in the same sense, then $\lambda/2$ will be the distance between two ether particles whose velocities of vibration are at any one instant equal but oppositely directed. Thus two trains of waves separated by an interval equal to λ are in perfect accord as to their vibrations; but when the distance between corresponding points is $\lambda/2$, then their vibrations are directly opposed. Accordingly

the above formula gives for the value of x, corresponding to the centre of the dark band of the first order, the following value: $\sqrt{\frac{\lambda b\,(a+b)}{a}}$; while observation shows that, as a matter of fact, this is the brightest part of the first fringe. On the same theory, the edge of the geometrical shadow, where the difference of path vanishes, ought to be brighter than the rest of the fringe, while, as a matter of fact, this is precisely the darkest region outside the geometrical shadow. In general, the position of the dark and bright bands deduced from this formula is almost exactly the inverse of that determined by experiment. This is the first difficulty presented by this theory. To avoid it, we must suppose that the rays reflected at the edge of the screen suffer the loss of half a wave-length; adding $\lambda/2$ to the difference of path, d, the general expression becomes

$$x=\sqrt{\frac{(2d+\lambda)\,b\,(a+b)}{a}}.$$

Replacing d in this formula by $\lambda/2$, $3\,\lambda/2$, $5\,\lambda/2$, $7\,\lambda/2$, etc., we have for the values of x corresponding to dark bands of the first, second, third, fourth, etc., orders:

$$\sqrt{\frac{2\lambda b\,(a+b)}{a}},\ \sqrt{\frac{4\lambda b\,(a+b)}{a}},\ \sqrt{\frac{6\lambda b\,(a+b)}{a}},\ \sqrt{\frac{8\lambda b\,(a+b)}{a}},\ \text{etc.}$$

These formulæ appear to agree fairly well with the observations; however, closer measurements show that the ratios between the sizes of the fringes derived from these expressions are not exactly correct, as we shall see later.

22. I pass now to the consideration of interior fringes produced in the shadow by the meeting of two pencils of light inflected at A and A′. Let M, Fig. 14, be any point located in the interior of the shadow; the intensity of the light at this point depends upon the amount of disagreement between the vibrations of the rays AM and A′M, which meet at this point, or upon the difference of path A′M − AM. I shall denote by x the distance MC of the point M from the middle of the shadow, and by d the difference of paths, and hence

$$d=\sqrt{b^2+(\tfrac{1}{2}c+x)^2}-\sqrt{b^2+(\tfrac{1}{2}c-x)^2}.$$

Expanding the radicals and neglecting the higher powers of x, since this quantity is very small compared with b, we have

$$d=cx/b,$$

or $$x = bd/c.$$

If in place of d in this expression we substitute successively $\lambda/2$, $3\lambda/2$, $5\lambda/2$, $7\lambda/2$, etc., we obtain the values of x corresponding to dark bands of the first, second, third, fourth, etc., order, namely,

$$\frac{b\lambda}{2c}, \quad \frac{3b\lambda}{2c}, \quad \frac{5b\lambda}{2c}, \quad \frac{7b\lambda}{2c}, \text{ etc.,}$$

and consequently for the distance between the middle points of two consecutive dark bands, $b\lambda/c$.

The general expression for n such intervals is, therefore, $nb\lambda/c$.

23. So long as the extreme fringes are sufficiently distant from the edges of the shadow, this formula agrees fairly well with experiment; but when they approach very near or pass beyond the edges, one detects a slight difference between their actual position and that deduced from the formula. In general, the calculated values are always a little larger than the observed. The reason for this I shall show when we come to the true theory of diffraction. It also follows from this formula that the size of the interior fringes ought to be entirely independent of the distance, a, of the luminous point from the opaque body; this prediction, however, is not completely verified by experiment, especially when the fringes completely fill the shadow; their position then varies distinctly with the distance a.

24. According to the formula

$$\sqrt{\frac{2n\lambda b\,(a+b)}{a}},$$

which we have just derived for the exterior fringes, their position depends upon a as well as upon b. Experiment shows that, in fact, their size increases or diminishes according as the opaque body approaches or recedes from the luminous point, and that the ratios between the different sizes of one and the same fringe deduced from the formula are precisely those given by observation. But the most remarkable inference from this formula is that, when a remains constant, the distance of any dark or bright band from the edge of the geometrical shadow is not directly proportional to b as in the case of interior fringes, but varies in such a way that this band traces out, not a straight line, but a hyperbola of sensible curvature. This is also confirmed by experiment, as may be seen from the observations given above.

Considering the striking agreement of these formulæ with experiment, it is natural to suppose that they are accurate expressions of fact, and therefore natural to attribute any small differences between calculated and observed values to the errors which are unavoidable in such delicate measurements.*

But a closer examination of the hypotheses from which they are derived, and of the inferences derivable from them, shows that they do not agree with the facts of nature.

25. If the fringes at the edge of a shadow are really due to the meeting of the direct rays with those reflected at the edge of the screen, their intensity ought to depend upon the area and the curvature of its surface, and the fringes produced by the back of a razor, for instance, ought to be much more vis-

* It might appear at first sight that one would be able to adapt this theory to the ideas of Newton by introducing the principle of interference, as I have indicated above; but besides the complication of fundamental hypotheses and the small probability of any of them, this principle, it appears to me, would lead to consequences which contradict the emission-theory.

M. Arago has remarked that the interposition of a thin transparent plate at the edge of an opaque body sufficiently narrow to produce interior fringes in its shadow displaces these fringes and shifts them towards the side [*see paper by Arago* (*Ann. Chim. et Phys.*, i., p. 199, 1816)] where is placed the transparent plate. This being so, it follows from the principle of interference that the rays which have traversed the plate have been retarded in their path, because the same fringes in each case must correspond to equal intervals between the times of arrival of rays. This inference at once confirms the wave-theory and manifestly contradicts the emission-theory, in which one is compelled to assume that light travels more rapidly in dense than in rare media.

This objection can be avoided only by substituting for difference of path difference of "fit"; but we lose all that was gained by the principle of interference in thus replacing a sharp idea by a hazy one, a satisfactory explanation by one which does not aid our understanding of the phenomena; for one can readily see how two light particles striking the retina at the same point may produce sensations more or less intense, according as the interval of time which separates two consecutive impacts is sufficient to produce unison or dissonance between the vibrations at the optic nerve; while it is by no means so easy to see how this effect could be produced by a difference of "fit" between two light particles, or how by simultaneous impact on the optic nerve they would produce no effect at all when they were in opposite "fits," even though their mechanical impacts were in perfect unison.

ible than those produced by the edge; but, using a magnifying-glass at a distance of some centimeters, one detects practically no difference in intensity in these two cases. This test is more easily made by using a steel plate one edge of which is round throughout a part of its length and sharp throughout the remainder of its length, these two edges lying in the same straight line. One is thus easily convinced that the fringes have the same intensity throughout their entire length.

26. We know that under large angles of incidence dull surfaces reflect light almost as well as polished mirrors. This is easily explained either on the emission-theory or on the wave-theory. But although one can understand how difference of polish cuts a small figure when the angle of incidence is large, it is not easy to see how the intensity of the reflected light can be independent of the curvature of the reflecting surface; indeed, it is clear that as the radius of curvature diminishes the reflected rays will diverge more and more, whatever be their angle of incidence.

27. Not only so, but I have convinced myself by another simple experiment of the incorrectness of the hypothesis which I had first adopted, and which I am now opposing. I cut a sheet of copper into the shape represented in Fig. 15, and placed it in a dark room about four meters in front of a luminous point, and examined its shadow with a magnifying-glass. What I observed, on slowly receding, was as follows: When the large fringes produced by each of the very narrow openings CEE′C′ and DFF′D′ had spread out into the geometrical shadow of CDFE, which received practically only white light from each separate slit, the interior fringes produced by the meeting of these two pencils of light showed colors much sharper and purer than the interior fringes of the shadow of ABDC, and were, at the same time, much brighter. On receding still farther, I noticed that the light diminished throughout the whole of the shadow of ABFE, but much more rapidly back of EFDC than in the upper part of the shadow, so that there was one particular instant when the intensity of the light ap-

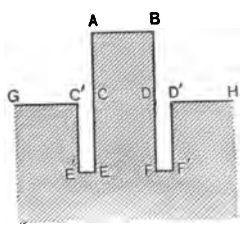


Fig 15.

peared to be the same above and below, after which the fringes remained less intense in the lower* part, although their colors were always much purer.

If, now, the only inflected light were that which grazed the edges of the opaque bodies, the fringes of the upper part ought to be sharper and ought to show purer colors than those of the lower part; for the first are produced by the meeting of two systems of waves which have their centres upon the edges AC and BD, while the others are formed by the meeting of four systems of waves having their origin at the edges C'E', CE, DF, DF'; and this would necessarily diminish the difference of intensity between the dark and bright bands, in the case of homogeneous light, or the purity of the colors, in the case of white light, because the fringes produced by the rays reflected and inflected at C'E' and DF would not exactly coincide with those produced by the meeting of rays coming from CE and D'F'. Now experiment shows, as I have just said, that exactly the reverse of this is true. One might explain on this same hypothesis how it happens that the shadow of ECDF is much brighter than that of ABDC arising from the double source of light presented by the two edges of each slit; but from this it would follow that the lower part ought always to be brighter, and we have just seen that this is not the fact.

28. From the experiments which I have just described it is evident that we cannot attribute the phenomena of diffraction solely to rays which graze the edge of the body; but we must, on the contrary, admit that there is an infinitude of other rays sensibly distant from the body and yet deviated from their original direction, so as to meet and form these fringes.

29. The spreading out of a pencil of light in passing through a very narrow opening shows in an even better manner that the inflection of light occurs at a sensible distance from the edges

* In order that this difference of intensity between the two parts of the shadow shall be as marked as possible, it is necessary that the slits CE and DF be very narrow as compared with the distance which separates them, and that the sheet of copper should be as far away as possible from the luminous point.

[*In repeating this experiment, it will be found very convenient to use, instead of sheet copper, an unfixed photographic plate: lantern slide is best. The two slits can be cut either with a pocket-knife or, better still, by means of a dividing engine.*]

of the diaphragm. It was in the consideration of this phenomenon that I discovered the error into which I had previously fallen. When one brings the edges of two opaque screens very close together in front of a luminous point in a dark room, he observes that the region illuminated by the aperture greatly increases. Such screens were Newton's two knife-edges. I shall suppose, as in his experiment, that the edges of the aperture are thin and perfectly sharp; not that this has any effect upon the phenomena, but simply for making clearer the conclusion which is to be drawn. The small number of rays which graze these sharp edges, being spread out over a rather large area, could produce only an insensible amount of illumination, or, at most, an exceedingly feeble light, and in the midst of it one ought to see a bright band traced out by the pencil of direct rays. This, however, is not the fact; for white light of almost uniform intensity fills a space much larger than the projection of the aperture,* and gradually grows weaker, shading into the dark bands of the first order. It was doubtless in order to account for the large amount of light inflected that Newton supposed the action of the body upon rays of light to extend to sensible distances, but this hypothesis will not bear careful scrutiny.

30. If the expansion of a pencil of light which passes through a narrow opening were brought about by attractive and repulsive forces having their origin at the edges of the aperture, the intensity of these forces, and consequently their effect upon the light, would *necessarily* vary with the nature, the mass, and the surface of the edges of the screen. All forces produced by a body acting at a sensible distance and taking their rise in any considerable extent of its mass or its surface would depend upon the relative positions and upon the number of particles contained within this sphere of activity, or, what is the same thing, upon the shape of the surface. If, then, the phenomena in question are due to the action of such forces, one would expect that, on placing a sharp body opposite a round body, the rays of light would be inflected more to the one side than the

* The illuminated space increases so rapidly in comparison with the width of the conical projection of the aperture as the receiving screen recedes from the aperture, and likewise when the aperture itself is further withdrawn from the luminous point, that by making these two distances sufficiently great one can obtain the same effect with an opening of any size.

other; but, as I have shown by a very simple experiment, this is not the fact. I passed a pencil of rays between two steel plates whose vertical edges were brought very close together and were carefully straightened throughout their entire length. A part of each edge was sharp, the rest of it round, and these edges were arranged so that the round portion of one plate corresponded to the sharp one of the other. Thus, if a sharp edge were located on the right in the upper part of the opening, another was located on the left in the lower part, so that if there had been any difference in the action of the two edges upon the rays, I should have noticed it in the relative positions of the upper and lower parts of the bright interval at the middle, and especially in the fringes in that neighborhood, as they would be interrupted at the point of passage from the sharp to the round edge; but, on observing them closely, I noted that they were perfectly straight throughout their entire length, even at the bright interval in the middle, exactly in the same way as when two edges of the same kind are opposed one to the other. The experiment may be varied by using plates made of two different substances, but the result* obtained will certainly remain the same.

31. All the experiments which I have tried so far have shown that the nature of the body interposed has in other respects no more influence upon the inflection of light than is exerted by the mass or the shape of the two edges. I shall cite only one experiment, in which I have taken every precaution necessary to determine the correctness of this principle, which, indeed, is already well established by the preceding experiment.

I covered an unsilvered mirror with a layer of India ink spread over a thin layer of paper, forming together a thickness of one-tenth of a millimeter. With a sharp point I traced two parallel lines, and then carefully removed from between these two lines the paper and the India ink which adhered to the surface of the glass. This aperture, as measured by the microm-

* Messrs. Berthollet and Malus found a long while ago that the nature of the body had no effect upon the diffraction of light. For screens they employed plates composed of different substances, with edges made up, for instance, of very dense metal and a piece of ivory; but they had no means of observation so convenient and accurate as mine, and consequently one might suspect that some small difference might have escaped them.

eter, was 1.17 mm. I then placed opposite each other two copper cylinders, each having a diameter of 14.5 mm., and by means of a graduated wedge I made the interval between these cylinders also 1.17 mm. The cylinders, placed alongside the blackened glass, were at a distance of 4.015 m. from the luminous point, and at a distance of 1.663 m. from the micrometer. I then measured the size of the fringes produced by these two openings, and found that they were absolutely the same. The following are the results of the two observations made with white light:

		mm.
The distance between the darkest point of the two dark bands of the first order at the point of separation of the brownish red from the violet .	First reading,	1.49
	Second reading,	1.49
The interval between the two fringes of the second order at the point of separation of red and green . . .	First reading,	3.22
	Second reading,	3.22

It is hardly possible that two sets of circumstances should differ more than these as regards the mass and the nature of the edges of the aperture. In the one case there is a single layer of India ink producing the fringes, for the glass to which it adheres completely fills the aperture; in the other case we have two massive cylinders of copper, 14.5 mm. in diameter, giving us an aperture whose edges have very considerable masses and areas, but we observe no difference in the expansion of the pencil of light.

32. It is therefore certain that the phenomena of diffraction do not at all depend upon the nature, the mass, or the shape of the body which intercepts the light,* but only upon the size of the intercepting body or upon the size of the aperture through which it passes. We must, therefore, reject any hypothesis which assigns these phenomena to attractive and repulsive

* This is so, at least provided one does not consider the shadow too close up to the edge of the screen, or provided the surface grazed by the rays of light has not too large an area compared with this distance; for in this case it may happen that the reflected rays sensibly affect the phenomenon —as, for instance, occurs when the surface grazed by the rays is a plane mirror of one or two decimeters in size and when one observes the fringes at a short distance. Besides, there would then be *successive* diffractions over an area too considerable for one to neglect.

forces whose action extends to a distance from the body as great as that at which rays are inflected. We are equally unable to admit that diffraction is caused by a shallow atmosphere which has the same thickness as the sphere of activity of these forces, and whose refractive index differs from that of the neighboring medium; for this second hypothesis, like the first, would lead us to think that the inflection of light ought to vary with the form and the nature of the edge of the screen, and ought not to be the same, for instance, at the edge and at the back of a razor. Now, on the emission-theory it is impossible to explain in any other manner the expansion of a beam of light passing through a narrow opening, and this expansion is a well-established fact.* Consequently, *the phenomena of diffraction cannot be explained on the emission-theory.*

SECTION II

33. In the first section of this mémoir I have shown that the corpuscular theory, and even the principle of interference when applied only to direct rays and to rays *reflected or inflected at the very edge of the opaque screen,* is incompetent to explain the phenomena of diffraction. I now propose to show that we may find a satisfactory explanation and a general theory in terms of waves, without recourse to any auxiliary hypothesis, by basing everything upon the principle of Huygens and upon that of interference, both of which are inferences from the fundamental hypothesis.

Admitting that light consists in vibrations of the ether similar to sound-waves, we can easily account for the inflection of rays of light at sensible distances from the diffracting body. For when any small portion of an elastic fluid under-

* The rise of a liquid in a capillary tube occurs between two surfaces separated by a finite distance, although the attraction which these surfaces exert upon the liquid extends only to an infinitely small distance. The reason of this is, that the molecules of the liquid, attracted by the surface of the tube, also in their turn attract other molecules of the liquid situated within their sphere of action, and so on, step by step; but in the emission-theory an analogous explanation is not admissible, for the fundamental hypothesis is that the luminous particles never exert any sensible effect upon the path of neighboring particles. No interdependence of motion is here admissible, for such an assumption would be the assumption of a fluid medium.

goes condensation, for instance, it tends to expand in all directions; and if throughout the entire wave the particles are displaced only along the normal, the result would be that all points of the wave lying upon the same spherical surface would simultaneously suffer the same condensation or expansion, thus leaving the transverse pressures in equilibrium; but when a portion of the wave-front is intercepted or retarded in its path by interposing an opaque or transparent screen, it is easily seen that this transverse equilibrium is destroyed and that various points of the wave may now send out rays along new directions.

To follow by analytical mechanics all the various changes which a wave-front undergoes from the instant at which a part of it is intercepted by a screen would be an exceedingly difficult task, and we do not propose to derive the laws of diffraction in this manner, nor do we propose to inquire what happens in the immediate neighborhood of the opaque body, where the laws are doubtless very complicated and where the form of the edge of the screen must have a perceptible effect upon the position and the intensity of the fringes. We propose rather to compute the relative intensities at different points of the wave-front only after it has gone a large number of wave-lengths beyond the screen. Thus the positions at which we study the waves are always to be regarded as separated from the screen by a distance which is very considerable compared with the length of a light-wave.

34. We shall not take up the problem of vibrations in an elastic fluid from the point of view which the mathematicians have ordinarily employed—that is, considering only a single disturbance. Single vibrations are never met with in nature. Disturbances occur in groups, as is seen in the pendulum and in sounding bodies. We shall assume that vibrations of luminous particles occur in the same manner—that is, one after another and series after series. This hypothesis follows not only from analogy, but as an inference from the nature of the forces which hold the particles of a body in equilibrium. To understand how a single luminous particle may perform a large series of oscillations all of which are nearly equal, we have only to imagine that its density is much greater than that of the fluid in which it vibrates—and, indeed, this is only what has already been inferred from the uniformity of the motions of

the planets through this same fluid which fills planetary space. It is not improbable also that the optic nerve yields the sensation of sight only after having received a considerable number of successive stimuli.

However extended one may consider systems of wave-fronts to be, it is clear that they have limits, and that in considering interference we cannot predicate of their extreme portions that which is true for the region in which they are superposed. Thus, for instance, two systems of equal wave-length and of equal intensity, differing in path by half a wave, interfere destructively only at those points in the ether where they meet, and the two extreme half wave-lengths escape interference.

Nevertheless, we shall assume that the various systems of waves undergo the same change throughout their entire extent, the error introduced by this assumption being inappreciable; or, what amounts to the same thing, we shall assume in our discussion of interference that these series of light-waves represent general vibrations of the ether, and are undefined as to their limits.

The Problem of Interference

35. *Given the intensities and relative positions of any number of trains of light-waves of the same length* and travelling in the same direction, to determine the intensity of the vibrations produced by the meeting of these different trains of waves, that is, the oscillatory velocity of the ether particles.*†

* We shall not here consider light-waves of different lengths which, in general, come from different sources and which cannot, therefore, give rise to simultaneous disturbances and cannot by their interaction produce any phenomena which are uniform; and even if they were uniform, the rise and fall of intensity produced by the interference of two different kinds of waves, after the manner of beats in sound, would be far too rapid to be detected, and would produce only a sensation of constant intensity.

† It was Mr. Thomas Young who first introduced the principle of interference into optics, where he showed much ingenuity in applying it to special cases; but in the problems which he has thus solved he has considered, I think, only the limiting cases, where the difference in phase between the two trains of waves is either a maximum or a minimum, and has not computed the intensity of the light for any intermediate cases or for any number whatever of trains of waves, as I here propose to do.

Employing the general principle of the superposition of small motions, the total velocity impressed upon any particle of a fluid is equal to the sum of the velocities impressed by each train of waves acting by itself. When these waves do not coincide, these different velocities depend not only upon the intensity of each wave, but also upon its phase at the instant under consideration. We must, therefore, know the law according to which the velocity of vibration varies in any one wave, and for this purpose we must trace the wave back to the origin whence it derives all its characteristics.

36. It is natural to suppose that the particles whose vibrations produce light perform their oscillations like those of sounding bodies—that is, according to the laws which hold for the pendulum; or, what is the same thing, to suppose that the acceleration tending to make a particle return to its position of equilibrium is directly proportional to the displacement. Let us denote this displacement by x. A suitable function of this displacement can then be represented by the expression $Ax+Bx^2+Cx^3+$etc., since this will vanish when $x=0$. If, now, we suppose the excursion of the particle to be very small when compared with the radius of the sphere throughout which the forces of attraction and repulsion act, we can neglect in comparison with Ax all other terms of the series and consider the acceleration as practically proportional to the distance x. This hypothesis, to which we are led by analogy, and which is the simplest that one can make concerning the vibrations of light particles, ought to lead to accurate results, since the laws of optics remain the same for all intensities of light.

Let us represent by v the velocity of vibration of a light particle at the end of a time t. We shall then have $dv=-Axdt$; but $v=dx/dt$, or $dt=dx/v$. Substituting in the first equation, we have $vdv=-Axdx$. Integrating, we have $v^2=C-Ax^2$; and hence

$$x=-\sqrt{\frac{C-v^2}{A}}.$$

Substituting this value of x in the first equation, we have

$$dt=\frac{dv}{\sqrt{A(C-v^2}},$$

which, on integration, gives

$$t=C'+\frac{1}{\sqrt{A}}\sin^{-1}\frac{v}{\sqrt{C}}.$$

If we measure time from the instant at which the velocity is zero, the constant C' becomes zero, and we have

$$t=\frac{1}{\sqrt{A}}\sin^{-1}\frac{v}{\sqrt{C}}, \text{ or } v=\sqrt{C}\sin t\sqrt{A}.$$

If we employ as unit of time the interval occupied by the particle in one complete vibration, we have $v=\sqrt{C}\sin(2\pi t)$. Thus, in isochronous vibrations, the velocities for equal values of t are always proportional to the constant $\sqrt{C}$, which, therefore, measures the intensity of the vibration.

37. Let us now consider the wave produced in the ether by the vibrations of this particle. The energy of motion in the ether at any point on the wave depends upon the velocity of the point-source at the instant when it started a disturbance which has just reached this point. The velocity of the ether particles at any point in space after an interval of time t is proportional to that of the point-source at the instant $t-x/\lambda$, x being the distance of this point from the source of motion and λ the length of a light-wave. Let us denote by u the velocity of the ether particles. We then have

$$u=a\sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right].$$

We know that the intensity a of vibration* [*oscillatory velocity*] in a fluid is in inverse ratio to the distance of the wave from the centre of disturbance; but, considering how minute these waves are when compared with the distance which separates them from the luminous point, we may neglect the variation of a and consider it as constant throughout the extent of one or even of several waves.

38. By the aid of this expression one can compute the intensity of vibration produced by the meeting of any number of pencils of light whenever he knows the intensity of the different trains of waves and their respective positions.

Let us first determine the velocity of a luminous particle in a vibration which results from the interference of two trains

* [*See last sentence of section 57 below.*]

of waves displaced, one with respect to the other, by a quarter of a wave-length [*i.e., differing in phase by* 90°], and having intensities which we shall denote by a and a'. We shall count time, t, from the moment at which the vibrations of the first train begin. Let u and u' be the velocities which the first and second trains of waves would impress upon a light particle whose distance from the source of motion is x. We then have

$$u=a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right] \text{ and } u'=a' \sin\left[2\pi\left(\frac{t-x+\frac{\lambda}{4}}{\lambda}\right)\right],$$

or

$$u'=-a' \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right].$$

Hence, the resultant velocity U will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-a' \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right].$$

Putting $a=A \cos i$ and $a'=A \sin i$, this expression may always be placed in the following form:

$$A \cos i \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-A \sin i \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

or

$$A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right].$$

Thus the wave produced by the meeting of two others will be of the same nature, but will have a different position [*phase*] and a different intensity. From the equations $A \cos i=a$ and $A \sin i=a'$, we have for the value of A (that is, for the intensity of the resultant wave) $\sqrt{a^2+a'^2}$; but this is exactly the value of the resultant of two mutually rectangular forces, a and a'.

From the same equations it is easily seen also that the new wave exactly corresponds in angular position [*phase*] to the resultant of the two mutually rectangular forces a and a'; for the equation

$$U=A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right]$$

shows that the linear displacement of this wave with respect to the first is $\frac{i\lambda}{2\pi}$; but i is also the angle which the force a

makes with the resultant A, because $A \cos \iota = a$. Thus we have complete analogy between the resultant of two mutually rectangular forces and the resultant of two trains of waves differing in phase by a quarter of a wave-length.

39. The solution of this particular case for waves differing by a quarter of a wave-length suffices to solve all other cases. In fact, whatever be the number of the trains of waves, and whatever be the intervals which separate them, we can always substitute for each of them its components referred to two reference points which are common to each train of waves and which are distant from each other by a quarter of a wave-length; then adding or subtracting, according to sign, the intensities of the components referred to the same point, we may reduce the whole motion to that of two trains of waves separated by the distance of a quarter of a wave-length; and the square root of the sum of the squares of their intensities will be the intensity of their resultant; but this is exactly the method employed in statics to find the resultant of any number of forces; here the wave-length corresponds to one circumference in the statical problem, and the interval of a quarter of a wave-length between the trains of waves to an angular displacement of 90° between the components.

40. It very often happens in optics that the intensities of light or the particular tint which one wishes to compute is produced by the meeting of only two trains of waves, as in the case of [*Newton's*] colored rings and the ordinary phenomena of color presented by crystalline plates. It is, therefore, well to know the general expression for the resultant of two trains of waves differing in phase by any amount whatever. The result is easily predicted from the general method which I have explained, but I think it will be wise to emphasize somewhat the theory of vibrations, and to show directly that the wave resulting from two others, separated by any interval whatever, corresponds exactly in intensity and position to the resultant of two forces whose intensities are equal to those of the two pencils of light, making an angle with each other which bears to one complete circumference the same ratio that the interval between the two trains of waves bears to one wave-length.

Let x be the distance from the origin of the first train of waves to the light particle under consideration, and t the instant for which we wish to compute its velocity. The speed impressed by the first train of waves will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

where a represents the intensity of this ray of light.

Let us call a' the intensity of the second pencil, and let us denote by c the distance between corresponding points on the two trains of waves; the [*oscillatory*] velocity due to the second train will then be

$$a' \sin\left[2\pi\left(t-\frac{x+c}{\lambda}\right)\right],$$

and hence the total velocity impressed upon the particle will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]+a' \sin\left[2\pi\left(t-\frac{x+c}{\lambda}\right)\right],$$

or

$$\left[a+a' \cos\left(2\pi\frac{c}{\lambda}\right)\right]\sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-a' \sin\left(2\pi\frac{c}{\lambda}\right)\cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

an expression to which may always be given the following form:

$$A \cos i \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-A \sin i \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

or

$$A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right],$$

where

$$a+a' \cos\left(2\pi\frac{c}{\lambda}\right)=A \cos i,$$

and

$$a' \sin\left(2\pi\frac{c}{\lambda}\right)=A \sin i.$$

Squaring and adding, we have

$$A^2=a^2+a'^2+2aa' \cos\left(2\pi\frac{c}{\lambda}\right).$$

Hence,

$$A=\pm\sqrt{a^2+a'^2+2aa' \cos\left(2\pi\frac{c}{\lambda}\right)}.$$

But this is precisely the value of the resultant of two forces, a and a', inclined to each other at an angle $2\pi\frac{c}{\lambda}$.

41. From this general expression it is seen that the resultant intensity of the light vibrations is equal to the sum of intensities of the two constituent pencils when they are in perfect agreement and to their difference when they are in exactly opposite phases, and, lastly, to the square root of the sum of their squares when their phase difference is a quarter of a wave-length, as we have already shown.

It thus follows that the phase of the wave corresponds exactly to the angular position of the resultant of two forces, a and a'. The distance from the first wave to the second is c, to the resultant wave $\frac{i\lambda}{2\pi}$, and from the resultant wave to the second is $c-\frac{i\lambda}{2\pi}$; accordingly, the corresponding angles are $2\pi.\frac{c}{\lambda}$, i, and $2\pi.\frac{c}{\lambda}-i$. Let us multiply the equation

$$a+a'\cos\left(2\pi\frac{c}{\lambda}\right)=A\cos i$$

by $\sin i$, and the following equation

$$a'\sin\left(2\pi\frac{c}{\lambda}\right)=A\sin i$$

by $\cos i$. Subtracting one from the other, we have

$$a\sin i=a'\sin\left(2\pi\frac{c}{\lambda}-i\right),$$

which, together with

$$a'\sin\left(2\pi\frac{c}{\lambda}\right)=A\sin i,$$

gives the following proportion:

$$\sin\left(2\pi\frac{c}{\lambda}-i\right):\sin i:\sin 2\pi\frac{c}{\lambda}::a:a':A.$$

42. The general expression, $A\sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right]$, for the velocity of the particles in a wave produced by the meeting of two others shows that this wave has the same length as its components and that the velocities at corresponding points are proportional, so that the resultant wave is always of the same nature as its components and differs only in intensity—that is to say, in the constant by which we must multiply the velocities in either of the components in order to obtain the correspond-

ing velocities in the resultant. In combining this resultant with still another new wave, one again arrives at an expression of the same form—a remarkable property of a function of this kind. Thus in the resultant of any number of trains of waves of the same length the light particles are always urged by velocities proportional to those of the components at points located at the same distance from the end of each wave. [*This is seen by multiplying each of the last three terms in the preceding proportion by* sin ωt. *For then,*

$$a \sin \omega t : a' \sin \omega t : A \sin \omega t :: a : a' : A' :: \textit{constant ratio.}]$$

Applications of Huygens's Principle to the Phenomena of Diffraction

43. Having determined the resultant of any number of trains of light-waves, I shall now show how by the aid of these interference formulæ and by the principle of Huygens alone it is possible to explain, and even to compute, all the phenomena of diffraction. This principle, which I consider as a rigorous deduction from the basal hypothesis, may be expressed thus: *The vibrations at each point in the wave-front may be considered as the sum of the elementary motions which at any one instant are sent to that point from all parts of this same wave in any one of its previous* positions, each of these parts acting independently the one of the other.* It follows from the principle of the superposition of small motions that the vibrations produced at any point in an elastic fluid by several disturbances are equal to the resultant of all the disturbances reaching this point at the same instant from different centres of vibration, whatever be their number, their respective positions, their nature, or the epoch of the different disturbances. This general principle must apply to all particular cases. I shall suppose that all of these disturbances, infinite in number, are of the same kind, that they take place simultaneously, that they

* I am here discussing only an infinite train of waves, or the most general vibration of a fluid. It is only in this sense that one can speak of two light waves annulling one another when they are half a wave-length apart. The formulæ of interference just given do not apply to the case of a single wave, not to mention the fact that such waves do not occur in nature.

are contiguous and occur in the single plane or on a single spherical surface. I shall make still another hypothesis with reference to the nature of these disturbances, *viz.*, I shall suppose that the velocities impressed upon the particles are all directed in the same sense, perpendicular to the surface of the sphere,* and, besides, that they are proportional to the compression, and in such a way that the particles have no retrograde motion. I have thus reconstructed a primary wave out of partial [*secondary*] disturbances. We may, therefore, say that the vibrations at each point in the wave-front can be looked upon as the resultant of all the secondary displacements which reach it at the same instant from all parts of this same wave in some previous position, each of these parts acting independently one of the other.

44. If the intensity of the primary wave is uniform, it follows from theoretical as well as from all other considerations that this uniformity will be maintained throughout its path, provided only that no part of the wave is intercepted or retarded with respect to its neighboring parts, because the resultant of the secondary displacements mentioned above will be the same at every point. But if a portion of the wave be stopped by the interposition of an opaque body, then the intensity of each point varies with its distance from the edge of the shadow, and these variations will be especially marked near the edge of the geometrical shadow.

Let C be the luminous point, AG the screen, AME a wave which has just reached A and is partly intercepted by the opaque body. Imagine it to be divided into an infinite number of small arcs—Am', $m'm$, mM, Mn, nn', $n'n''$, etc. In order to determine the intensity at any point P in any of the later positions of the wave BPD, it is necessary to find the resultant of

* It is possible for light-waves to occur in which the direction of the absolute velocity impressed upon the particles is not perpendicular to the wave surface. In studying the laws of interference of polarized light, I have become convinced since the writing of this memoir that light vibrations are at right angles to the rays or parallel to the wave surface. The arguments and computations contained in this memoir harmonize quite as well with this new hypothesis as with the preceding, because they are quite independent of the actual direction of the vibrations and pre-suppose only that the direction of these vibrations is the same for all rays belonging to any system of waves producing fringes.

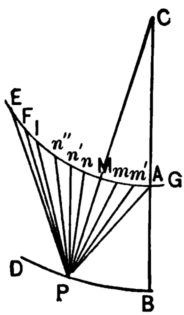


Fig. 16

all the secondary waves which each of these portions of the primitive wave would send to the point P, provided they were acting independently one of the other.

Since the impulse communicated to every part of the primitive wave was directed along the normal, the motion which each [*part of the wave*] tends to impress upon the ether ought to be more intense in this direction than in any other; and the rays which would emanate from it, if acting alone, would be less and less intense as they deviated more and more from this direction.

45. The investigation of the law according to which their intensity varies about each centre of disturbance is doubtless a very difficult matter;* but, fortunately, we have no need of knowing it, for it is easily seen that the effects produced by these rays are mutually destructive when their directions are sensibly inclined towards the normal. Consequently, the rays which produce any appreciable effect upon the quantity of light received at any point P may be regarded as of equal intensity.†

Let us now consider the rays EP, FP, and IP, which are sen-

* [*This is the problem solved by Stokes*; *Math. and Phys. Papers*, vol. ii., p. 243.]

† When the centre of disturbance has been compressed, the force of expansion tends to thrust the particles in all directions; and if they have no backward motion, the reason is simply that their initial velocities forward destroy those which expansion tends to impress upon them towards the rear; but it does not follow that the disturbance can be transmitted only along the direction of the initial velocities, for the force of expansion in a perpendicular direction, for instance, combines with a primitive impulse without having its effect diminished. It is clear that the intensity of the wave thus produced must vary greatly at different points of its circumference, not only on account of the initial impulse, but also because the compressions do not obey the same law around the centre of disturbance; but the variations of intensity in the resultant wave must follow the law of continuity, and may, therefore, be considered as vanishing throughout a small angle, especially along the normal to the primitive wave. For the initial velocities of the particles in any direction whatever are proportional to the cosine of the angle which this direction makes with that of the normal, so that these components vary much less rapidly than the angle so long as the angle is small.

sibly inclined and which meet at P, a point whose distance from the wave EA I shall suppose to include a large number of wave-lengths. Take the two arcs EF and FI of such a length that the differences EP—FP and FP—IP shall be equal to a half wave-length. Since these rays are quite oblique, and since a half wave-length is very small compared with their length, these two arcs will be very nearly equal, and the rays which they send to the point P will be practically parallel; and since corresponding rays on the two arcs differ by half a wave-length, the two are mutually destructive.

We may then suppose that all the rays which various parts of the primary wave AE send to the point P are of equal intensity, since the only rays for which this assumption is not accurate produce no sensible effect upon the quantity of light which it receives. In the same manner, for the sake of simplifying the calculation of the resultant of all the elementary waves, we may consider their vibrations as taking place in the same direction, since the angles which these rays make with each other are very small; so that the problem reduces itself to the one which we have already solved—namely, *to find the resultant of any number of parallel trains of light-waves of the same length, the intensities and relative positions being given.* The intensities are here proportional to the lengths of the illuminating arcs, and the relative positions of the wave trains are given by the differences of path traversed.

46. Properly speaking, we have considered up to this point only the section of the wave made by a plane perpendicular to the edge of the screen projected at A. We shall now consider it in its entirety, and shall think of it as divided by equidistant meridians perpendicular to the plane of the figure into infinitely thin spindles. We shall then be able to employ the same process of reasoning which we have just used for a section of the wave, and thus show that the rays which are quite oblique are mutually destructive.

In the case we are now considering these spindles are indefinitely extended in a direction parallel to the edge of the screen, for the wave is intercepted only on one side. Accordingly the intensity of the resultant of all the vibrations which they send to the point P would be the same for each of them; for, owing to the extremely small difference of path, the rays which emanate from these spindles must be considered as of equal intensity, at least throughout that region of the primitive wave which produces a sensible effect upon the light sent to P. Further, it is evident that each elementary resultant will differ in phase by the same quantity with respect to the ray coming from that point of the spindle nearest P, that is to say, from the point at which the spindle cuts the plane of the figure. The intervals between these elementary resultants will then be equal to the difference of path traversed by the rays AP, m'P, mP, etc., all lying in the plane of the figure; and their intensities will be proportional to the arcs Am', $m'm$, mM, etc. In order now to obtain the intensity of the total resultant, we have to make the same calculation which we have already made, considering only the section of the wave by a plane perpendicular to the edge of the screen.*

47. Before deriving the analytical expression for this resultant I propose to draw from the principle of Huygens some of the inferences which follow from simple geometrical considerations.

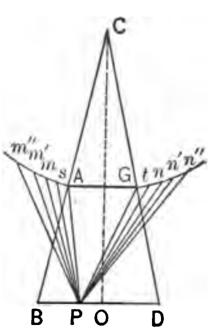


Fig. 17

Let AG represent an opaque body sufficiently narrow for one to distinguish fringes in its shadow at the distance AB. Let C be the luminous point and BD be either the focal plane of the magnifying-glass with which one observes these fringes or a white card upon which the fringes are projected.

Let us now imagine the original wave divided into small arcs—Am, mm', $m'm''$, etc., Gn, nn', $n'n''$, $n''n'''$, etc.—in such a way that the rays drawn from the point P in the shadow to two consecutive points of division will differ by half a wave-length. All of the secondary waves sent to the point P by the elements of each of these arcs will completely interfere with those which emanate

* So long as the edge of the screen is rectilinear we can determine the position of the dark and bright bands and their relative intensities by considering only the section of the wave made by a plane which is perpendicular to the edge of the screen. But when the edge of the screen is curved or composed of straight edges inclined at an angle it is then necessary to integrate along two directions at right angles to each other, or to integrate around the point under consideration. In some particular cases this latter method is simpler, as, for instance, when we have to calculate the intensity of the light in the centre of the shadow produced by a screen or in the projection of a circular aperture.

from the corresponding parts of the two arcs immediately adjoining it; so that, if all these arcs were equal, the rays which they would send to the point P would be mutually destructive, with the exception of the extreme arc *m*A. Half of the intensity of this arc would be left, for half the light sent by the arc *mm'* (with which *m*A is in complete discordance) would be destroyed by half of the preceding arc *m''m'*. As soon as the rays meeting at P are considerably inclined with respect to the normal, these arcs are practically equal. The resultant wave, therefore, corresponds in phase almost exactly to the middle of *m*A, the only arc which produces any sensible effect. It is thus seen that it differs in phase by one-quarter of a wave-length from the element at the edge A of the opaque screen. Since the same thing takes place in the other part of the incident wave G*n*, the interference between these two vibrations occurring at the point P is determined by the difference of length between the two rays *s*P and *t*P, which take their rise at the middle of the arcs A*m* and G*n*, or, what amounts to the same thing, by the difference between the two rays AP and GP coming from the very edge of the opaque body. It thus happens that when the interior fringes under consideration are rather distant from the edges of the geometrical shadow, we are able to apply practically without error the formula based upon the hypothesis that the inflected waves have their origin at the very edges of the opaque body; but in proportion as the point P approaches B the arc A*m* becomes greater in comparison with the arc *mm'*, the arc *mm'* with respect to the arc *m'm''*, etc.; and likewise in the arc *m*A the elements in the immediate vicinity of the point A become sensibly greater than the elements which are situated near the point *m*, and which correspond to equal differences of path. It happens, therefore, that the *effective** ray, *s*P, will not be the mean between the outside rays, *m*P and AP, but will more nearly approach the length of the latter. On the other side of the opaque body we have slightly different circumstances. The difference between the ray GP and the effective ray *t*P approximates more and more nearly a quarter of a wave-length

* I have given this name to the distance of the resultant wave from the original wave because the positions of the dark and bright bands are the same as they would be if these effective rays alone produced them.

as the point P moves farther and farther away from D, so that the difference of path traversed varies more rapidly between the effective rays sP and tP than between the rays AP and GP; consequently, the fringes in the neighborhood of the point B ought to be a little farther from the centre of the shadow than would be indicated by the formula based upon the first hypothesis.

48. Having considered the case of fringes produced by a narrow body, I pass to the consideration of those which are caused by a small aperture.

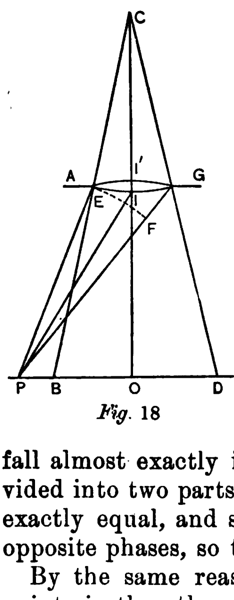


Fig. 18

Let AG be the aperture through which the light passes. I shall at first suppose that it is sufficiently narrow for the dark bands of the first order to fall inside the geometrical shadow of the screen, and at the same time to be fairly distant from the edges B and D. Let P be the darkest point in one of these two bands; it is then easily seen that this must correspond to a difference of one whole wave-length between the two extreme rays AP and GP. Let us now imagine another ray, PI, drawn in such a way that its length shall be a mean between the other two. Then, on account of its marked inclination to the arc AIG, the point I will fall almost exactly in the middle. We now have the arc divided into two parts, whose corresponding elements are almost exactly equal, and send to the point P vibrations in exactly opposite phases, so that these must annul each other.

By the same reasoning it is easily seen that the darkest points in the other dark bands also correspond to differences of an even number of half wave-lengths between the two rays which come from the edges of the aperture; and, in like manner, the brightest points of the bright bands correspond to differences of an uneven number of half wave-lengths—that is to say, their positions are exactly reversed as compared with those which are deduced from the interference of the limiting rays on the hypothesis that these alone are concerned in the production of fringes. This is true with the exception of the point at the middle, which, on either hypothesis, must be bright. The inferences deduced from the theory that the fringes result from the superposition of all of the disturbances from all parts of the arc AG are verified by experiments, which at the same time disprove the theory which looks upon these bands as produced only by rays inflected and reflected at the edges of the diaphragm. These are precisely the phenomena which first led me to recognize the insufficiency of this hypothesis, and suggested the fundamental principle of the theory which I have just explained—namely, the principle of Huygens combined with the principle of interference.

49. In the case which we have just considered, where, by virtue of a very small aperture, the dark bands of the first order fall at some distance from the edges of the geometrical shadow, it follows from theory, as well as from experiment, that the distance comprised between the darkest points is almost exactly double that of the other intervals between the middle points of two consecutive dark bands, and this is all the more nearly true in proportion as the aperture becomes smaller or more distant from the luminous point and from the focus of the magnifying-glass with which one observes the fringes; for, by sufficiently increasing these distances one may produce the same effects with an aperture of any size whatever.

But when these distances are not very great, and when their aperture is too large for the rays producing the fringes to be very much inclined to the wave-front, AG, it follows that corresponding elements of the arcs into which we have supposed a wave to be divided can no longer be considered as each equal to the other, for they are sensibly larger on the side next the band under consideration. Under these conditions we can rigorously deduce the positions of maximum and minimum intensity only by computing the resultant of all the small secondary waves which are sent out by the incident wave.

50. But there is one very remarkable case where a knowledge of this integral is not needed for the determination of the law of the fringes by an aperture of very considerable size. This is the case where a lens is placed in front of the diaphragm, and brings the refracted rays to focus upon the plane in which the fringes are observed. The problem is now greatly simplified by the fact that the centre of curvature of the

emergent wave now lies in this plane instead of at the luminous point.

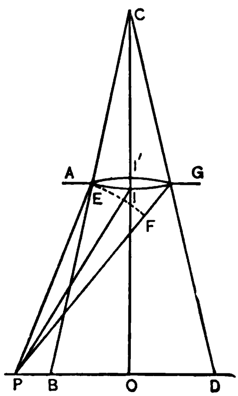


Fig. 18

Let O be the projection of the middle point of the aperture upon this plane. From the point O as centre, and with a radius equal to AO, let us now describe the arc AI'G, which will now represent the incident wave as modified by the interposition of the lens. If, now, from the point P as centre, and with a radius AP, we describe the arc AEF, those portions of the luminous rays meeting at the point P which are comprised between the arc AI'G and the arc AEF will be the differences of path traversed by the secondary waves; and, since these two arcs have equal curvatures and are convex towards the same side, it follows that equal differences of path will correspond to equal intervals upon the wave-front AI'G. Let us suppose this wave divided in such a manner that any two consecutive rays drawn through the points of division shall differ by one-half a wave-length. If, then, the point P be located in such a way that the total number of these arcs is even, it will no longer receive any light. For these arcs, taken two and two, are mutually destructive, since the vibrations due to corresponding elements are at the same time of equal intensity and opposite phase. The light reaching any point P will be a maximum when the total number of arcs is uneven. The brightest points of the bright bands, therefore, correspond to a difference of an uneven number of half wave-lengths between the two rays coming from the edges of the diaphragm, and the darkest points on the dark bands to a difference of an even number of half wave-lengths. Consequently, all the dark bands will be equally spaced among themselves, with the exception of the first two, where the interval is exactly double that which separates the others. This result, which had already been suggested by theory, I found to be thoroughly confirmed by experiment. I shall cite only one experiment of this kind made in homogeneous red light. In order to bring the centre of the incident wave to the plane of the micrometer wire, I used, instead of an ordinary lens, a glass cylinder, which, in order to get the full length of the fringes, I placed with its generating line parallel to the edges of the aperture in the diaphragm.

	mm.
Size of the aperture	2.00
	m.
Distance from the luminous point to the diaphragm, or *a*	2 507
Distance from the diaphragm to the micrometer, or *b*	1.140
	mm.
Interval between the middle points of the two dark bands of the first order	0.72
Interval between the band of the first order and the third	0.73
Interval between the band of the third order and the fifth	0.72

It will be observed that the first interval is double that of the others.

I have observed that the same law holds, even at distances which are not very great, for apertures which are much wider, a centimeter or even a centimeter and a half; but if we further increase the aperture of the diaphragm, the fringes become confused, however much care be taken to place the micrometer in the focus of the cylindrical lens; which goes to show that the rays refracted by this glass vibrate in unison [*in the same phase*] only within rather narrow limits, just as happens with ordinary lenses.

51. When the aperture of the diaphragm thus backed with a cylindrical lens is not too great, the dark and bright bands produced are as sharp as the fringes which result from the union of rays reflected from two mirrors. But, in the latter case, the intensity of the light is the same for all fringes, or, at least, whatever differences there are appear to arise merely from the fact that the light employed is not perfectly homogeneous; and if it happen that the bright bands diminish in brilliancy, the dark bands become less dark, so that the sum of the light in one entire fringe remains practically constant. But in the other phenomenon, as one recedes from the centre he observes a rapid diminution of the light, which is easily accounted for by the theory we have just explained. For, indeed, all the rays which leave the wave-front AI'G and meet at the centre of the bright band of the first order have traversed equal paths; so that all the small secondary waves which they bring to this point coincide [*in phase*] and strengthen each other.

But this is not the case with the other bright bands. The brightest band of the second order, for instance, corresponds to a division of the wave AI'G into three arcs, the extreme rays of which differ by one-half a wave-length; the effects produced by two of these arcs annul each other. Consequently, this band receives light from only one-third of the incident wave-front, while even the effect produced by this third is somewhat diminished by the fact that there is a difference of one-half a wave-length between the rays from its edges. A similar process of reasoning shows that the middle of the bright band of the third order is illuminated by only one-fifth of the wave-front AI'G, the light of this one-fifth being still further diminished by opposition of phase in its extreme rays.

[*Here are omitted six pages, including a geometrical discussion of the general relations between size of aperture* (*or obstacle*), *distance of screen, distance of luminous point, etc.*]

56. I have just explained the general relations between the size of any particular fringe and the respective distances of the obstacle from the luminous point and from the micrometer. As we have seen, these laws may be derived from theory quite independently of any knowledge of the integral which at each point represents the resultant of all the secondary waves; but in order to find the absolute size of these fringes, it is essential that we compute this resultant, for the positions of maxima and minima of intensity can be determined only by a comparison of the different values of this resultant, or at least by knowing the function which represents it.

In order to do this, we propose to apply to the principle of Huygens the method which we have already explained for computing the resultant of any number of trains of waves when their intensities and relative positions [*phases*] are given.

Application of Theory of Interference to Huygens's Principle

57. Let the waves from any luminous point C be partly intercepted by an opaque body AG. To begin with, we shall suppose that this screen is so large that no light comes around the edge G, so that we need consider only that part of the wave which lies to the left of the point A. Let DB represent the

plane upon which are received the shadow and its fringes. The problem then is to find the intensity of the light at any point P in this plane.

If from C as centre and with a radius CA we describe the circle AMI, it will represent the light-wave at the instant it is partly intercepted by the opaque body. It is from this position of the wave that I have computed the resultant of the secondary waves sent to the point P. If we consider the wave in an earlier position, say A′M′I′, it then becomes necessary to calculate the effect of the obstacle on each of the secondary waves arising from the arc A′M′I′; and if we consider the wave in a later position, say A″M″I″, it becomes necessary to first determine the intensities of its various points, for they are no longer equal, having been changed by the interposition of the screen. In this case the computation is vastly more complicated, possibly quite impracticable. If, however, we consider the wave at the instant it reaches A, the process is simple; for then all parts of the wave have the same intensity. Not only so, but none of the secondary waves are now affected by the opaque screen. However numerous the subdivisions into which we may consider these elementary waves divided, it is evident that the number will be the same for each, since they are transmitted freely in all directions. And, therefore, we need only consider the axes of these pencils of split rays—*i. e.*, the straight lines drawn from the various points on the wave AMI to the point P. The differences of length in these direct rays are the differences of path traversed by the elementary or partial resultants meeting at P.*

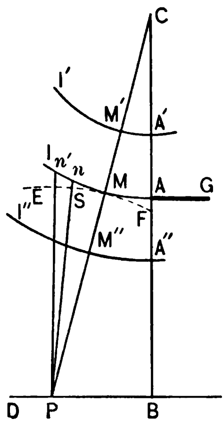


Fig. 19

In order to compute the total effect, I refer these partial resultants to the wave emitted by the point M on the straight line CP, and to another wave displaced a quarter of a wave-length with reference to the preceding. This is the process already employed (p. 101) in the general solution of the interference

* [*A foot-note is here omitted.*]

problem. We shall consider only a section of the wave made by a plane perpendicular to the edge of the screen, and shall indicate by dz an element, nn', of the primary wave, and by z its distance from the point M. These, as I have shown, suffice to determine the position and the *relative* intensities of the bright and dark bands. The distance nS included between the wave AMI and the tangential arc, EMF, described about the point P as centre is $\frac{1}{2}\frac{z^2(a+b)}{ab}$, where a and b are, as before, the distances CA and AB. If we denote the wave-length by λ, we have for the component in question, referred to the wave leaving the point M, the following expression

$$dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right);$$

while for the other component,* referred to a wave displaced a quarter of a wave-length from the first, we have

$$dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right).$$

If, now, we take the sum of all similar components of all the other elements, we shall have

$$\int dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right) \text{ and } \int dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right).$$

Hence the intensity of the vibration at P resulting from all these small disturbances is

$$\sqrt{\left[\int dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)\right]^2 + \left[\int dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)\right]^2}.$$

The intensity of the sensation, being proportional to the square of the speeds of the particles, is

$$\left[\int dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)\right]^2 + \left[\int dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)\right]^2.$$

This is what I have called the *intensity of the light* in order to conform to ordinary usage, while reserving the expression *intensity of vibration* to designate the speed of an ether particle during its oscillation.

* [*These expressions for amplitude follow directly from sec.* 40, *when in the general expression for velocity we make* $a=o$, $a'=dz$, *and* $c=\frac{1}{2}\frac{z^2(a+b)}{ab}$.]

58. In the case we are now considering, where the body, AG, is so large that we can neglect any light coming around the edge G, the integration extends from A to infinity on the side towards I. This integral naturally divides into two parts, one extending from A to M, the other from M to infinity. This latter integral remains constant, while the former varies with the position of the point P. This variation, indeed, is the determining factor in the size and relative intensity of the bright and dark bands.

The integrals

$$\int dz \cos\left(\pi\frac{z^2(a+b)}{ab\lambda}\right) \text{ and } \int dz \sin\left(\pi\frac{z^2(a+b)}{ab\lambda}\right)$$

may be evaluated in finite terms when the limits of z are taken at zero and infinity; but between any other limits their values can be expressed only in terms of a series or by means of partial integration.

The latter method seems to me more convenient, and I have, therefore, employed it in the computation of the following table, where the limits of integration are taken so close together that we can neglect the square of half the arc included between them.*

* Let i and $i+t$ be the narrow limits between which it is proposed to integrate $dv \cos qv^2$ and $dv \sin qv^2$. Neglecting the square of $\frac{t}{2}$, we then find the following approximate values for these integrals:

$$\int dv \cos qv^2 = \frac{1}{2q\left(i+\frac{t}{2}\right)}\left[\sin q\left(i+\frac{t}{2}\right)\left(i+\frac{3t}{2}\right) - \sin q\left(i+\frac{t}{2}\right)\left(i-\frac{t}{2}\right)\right]$$

$$\int dv \sin qv^2 = \frac{1}{2q\left(i+\frac{t}{2}\right)}\left[-\cos q\left(i+\frac{t}{2}\right)\left(i+\frac{3t}{2}\right) + \cos q\left(i+\frac{t}{2}\right)\left(i-\frac{t}{2}\right)\right].$$

These are the formulæ which I have used in the computation of the table. When the limits are sufficiently narrow for us to neglect t^2 instead of $\left(\frac{t}{2}\right)^2$, the following still simpler formulæ may be employed:

$$\int_{i+t}^{i} dv \cos qv^2 = \frac{1}{2iq}\left[\sin qi\,(i+2t) - \sin qi^2\right]$$

$$\int_{i+t}^{i} dv \sin qv^2 = \frac{1}{2iq}\left[-\cos qi\,(i+2t) + \cos qi^2\right].$$

This arc here amounts to $\frac{1}{10}$ of a quadrant, since this furnishes results of an accuracy greater than is attainable in the observations. In place of the integrals mentioned above, I have substituted $\int dv \sin qv^2$ and $\int dv \cos qv^2$, where q stands for quadrant or $\frac{\pi}{2}$.

To pass from one of these forms to the other is a simple matter.

[Following is a derivation of these formulæ which Verdet found in one of Fresnel's journals.

Let the limits of integration be denoted by a and $a+2p$.

Put $v=a+p+u$. Then $du=dv$; and when $v=a$, $u=-p$; but when $v=a+2p$, $u=+p$.

Substituting for v,

$$\int dv \cos qv^2 = \int du \cos q\left[u^2+2(a+p)u+(a+p)^2\right].$$

If the limiting values of u are $+p$ and $-p$, we may take p so small, say $\frac{1}{10}$, that we may neglect its square, u^2. We then have

$$\int_{a+2p}^{a} dv \cos qv^2 = \int_{-p}^{+p} du \cos q\left[2u(a+p)+(a+p)^2\right]$$

$$=\left|_{-p}^{+p}\frac{1}{2q(a+p)}\sin q\left[2u(a+p)+(a+p)^2\right]\right.$$

$$=\frac{1}{2q(a+p)}\left\{\sin q(a+p)(a+3p)-\sin q(a+p)(a-p)\right\}$$

TABLE OF THE NUMERICAL VALUES OF THE INTEGRALS

$\int dv \cos qv^2$ and $\int dv \sin qv^2$.*

Limits of Integrals	$\int dv \cos qv^2$	$\int dv \sin qv^2$	Limits of Integrals	$\int dv \cos qv^2$	$\int dv \sin qv^2$
From $v=0^q$			From $v=0^q$		
to $v=\overset{q}{0}.10$	0.0999	0.0006	to $v=\overset{q}{2}.90$	0.5627	0.4098
to $v=0.20$	0.1999	0.0042	to 3.00	0.6061	0.4959
0.30	0.2993	0.0140	3.10	0.5621	0.5815
0.40	0.3974	0.0332	3.20	0.4668	0.5931
0.50	0.4923	0.0644	3.30	0.4061	0.5191
0.60	0.5811	0.1101	3.40	0.4388	0.4294
0.70	0.6597	0.1716	3.50	0.5328	0.4149
0.80	0.7230	0.2487	3.60	0.5883	0.4919
0.90	0.7651	0.3391	3.70	0.5424	0.5746
1.00	0.7803	0.4376	3.80	0.4485	0.5654
1.10	0.7643	0.5359	3.90	0.4226	0.4750
1.20	0.7161	0.6229	4.00	0.4986	0.4202
1.30	0.6393	0.6859	4.10	0.5739	0.4754
1.40	0.5439	0.7132	4.20	0.5420	0.5628
1.50	0.4461	0.6973	4.30	0.4497	0.5537
1.60	0.3662	0.6388	4.40	0.4385	0.4620
1.70	0.3245	0.5492	4.50	0.5261	0.4339
1.80	0.3342	0.4509	4.60	0.5674	0.5158
1.90	0.3949	0.3732	4.70	0.4917	0.5668
2.00	0.4886	0.3432	4.80	0.4340	0.4965
2.10	0.5819	0.3739	4.90	0.5003	0.4347
2.20	0.6367	0.4553	5.00	0.5638	0.4987
2.30	0.6271	0.5528	5.10	0.5000	0.5620
2.40	0.5556	0.6194	5.20	0.4390	0.4966
2.50	0.4581	0.6190	5.30	0.5078	0.4401
2.60	0.3895	0.5499	5.40	0.5573	0.5136
2.70	0.3929	0.4528	5.50	0.4785	0.5533
2.80	0.4678	0.3913			

* From the text, and also from the first column of this table, one would be led to think that the second and third columns in the table give the values of the integrals $\int_0^v dv \cos \frac{\pi}{2} v^2$ and $\int_0^v dv \sin \frac{\pi}{2} v^2$ for the following values of v:

$$v=\frac{1}{10}\frac{\pi}{2},$$

$$v=\frac{2}{10}\frac{\pi}{2},$$

$$v=\frac{3}{10}\frac{\pi}{2},$$

etc.

That this is not the case, however, may be shown by using the approximation formulæ of Fresnel to compute any pair of consecutive values of

Either of the integrals $\int dv \cos qv^2$ and $\int dv \sin qv^2$ taken from zero to infinity have the value $\frac{1}{2}$. We may thus by the aid of the above table find the intensity of light corresponding to any given position of the point P, or, what is the same thing, corresponding to any definite value of v, where v is one limit of integration and infinity the other. We have only to take from the table the values of $\int dv \cos qv^2$ and $\int dv \sin qv^2$, using the value of v as an argument, then add to each $\frac{1}{2}$, and finally take the sum of their squares.

59. Simple inspection of this table shows a periodic change in the intensity of light as one leaves the geometrical shadow. To obtain the values of v corresponding to *maxima* and *minima*, *i. e.*, the brightest and darkest points in the respective bright and dark bands, I take from the table the numbers which most nearly correspond to them and then compute the corresponding intensities. Finally, by means of these data and a simple formula of approximation, I determine with sufficient accuracy the values of v which give *maxima* and *minima*.

Let us represent by i the approximate value of v taken directly from the table, by I and Y the corresponding values of $\frac{1}{2}+\int dv \cos qv^2$ and $\frac{1}{2}+\int dv \sin qv^2$, and by t the small arc by which v must be increased in order to give the *maximum* or *minimum* of light. Neglecting the square of t, we find that the following formula gives the value of t which yields a *maximum* or a *minimum*.

$$\text{Sin}\left[q\left(i^2+2it\right)\right]=\frac{2qiI-\sin qi^2}{\sqrt{(qiI-\sin qi^2)^2+(2qiY+\cos qi^2)^2}}.$$

[*A foot-note containing the derivation of this expression is here omitted.*]

If in this formula we substitute the numbers taken from the table, we obtain the following results :

either integral. The successive values of v employed in the first column are

$v=0.1$,
$v=0.2$,
$v=0.3$,
etc.

The same remark applies to the following tables. [*E. Verdet.*]

TABLE OF MAXIMA AND MINIMA FOR EXTERIOR FRINGES AND OF THE CORRESPONDING INTENSITIES

	Values of V	Intensities of Light
Maximum of 1st order.........	1.2172	2.7413
Minimum of 1st order.........	1.8726	1.5570
Maximum of 2d order.........	2.3449	2.3990
Minimum of 2d order.........	2.7392	1.6867
Maximum of 3d order.........	3.0820	2.3022
Minimum of 3d order.........	3.3913	1.7440
Maximum of 4th order.........	3.6742	2.2523
Minimum of 4th order.........	3.9372	1.7783
Maximum of 5th order.........	4.1832	2.2206
Minimum of 5th order.........	4.4160	1 8014
Maximum of 6th order.........	4.6369	2.1985
Minimum of 6th order.........	4.8479	1.8185
Maximum of 7th order.........	5.0500	2.1818
Minimum of 7th order.........	5.2442	1.8317

It is to be observed that here none of the *minima* become zero, as in the case of Newton's rings, or in fringes produced by the meeting of two beams of light of equal intensities ; here the difference between *maxima* and *minima* diminishes as one goes farther away from the edge of the opaque screen.

This explains why the fringes which border shadows are not so bright or so numerous as the colored rings, or as the bands produced by the reflection of a luminous point in two slightly inclined mirrors.

60. To employ the above table in computing the size of the exterior fringes, we must first recall the substitution of the integrals $\int dv \cos qv^2$ and $\int dv \sin qv^2$ for the integrals * in question,

$$\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right) \text{ and } \int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right);$$

whence

$$2q\frac{z^2(a+b)}{ab\lambda}=qv^2,$$

and

$$z=v\sqrt{\frac{ab\lambda}{2(a+b)}}.$$

* [*In what follows Fresnel replaces* $\frac{\pi}{2}$ *by* q. *the initial letter of "quadrant."*]

Therefore

$$\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right)=\sqrt{\frac{ab\lambda}{2(a+b)}}\int dv \cos qv^2,$$

and

$$\int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right)=\sqrt{\frac{ab\lambda}{2(a+b)}}\int dv \sin qv^2.$$

Also

$$\left[\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right)\right]^2+\left[\int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right)\right]^2$$

$$=\frac{ab\lambda}{2(a+b)}\left[\left(\int dv \cos qv^2\right)^2+\left(\int dv \sin qv^2\right)\right]^2.$$

Now the factor $\frac{ab\lambda}{2(a+b)}$ is constant; whence we infer that these two quantities,

$$\left[\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right)\right]^2+\left[\int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right)\right]^2$$

and

$$\left(\int dv \cos qv^2\right)^2+\left(\int dv \sin qv^2\right)^2,$$

will each reach their maximum or minimum values at the same time. Let us now denote by n the value of v which yields a maximum or minimum value for these integrals; the corresponding value of z will then be

$$z=n\sqrt{\frac{ab\lambda}{2(a+b)}}.$$

The size of the fringe, x, [*that is, its distance from the edge of the opaque screen*] then follows from the proportion

$$a:z::a+b:x;$$

whence

$$x=\frac{z(a+b)}{a};$$

or, substituting for z,

$$x=n\sqrt{\frac{\frac{1}{2}(a+b)b\lambda}{a}}.$$

This radical, it may be remarked, is exactly the distance between the edge of the geometrical shadow and that point which corresponds to a difference* of a quarter of a wave-length between the direct ray and the ray coming *via* the edge of the opaque screen. But this is precisely what might have been predicted, inasmuch as the corresponding value of v [*viz., the quadrant*] has been taken as unity in the table of numerical values of $\int dv \cos qv^2$ and $\int dv \sin qv^2$.

If in the formula

$$x = n\sqrt{\frac{(a+b)b\lambda}{2a}}$$

we substitute for n the value corresponding to a minimum of the first order—*i. e.*, to the darkest part of the first dark band, we have

$$x = 1.873\sqrt{\frac{(a+b)b\lambda}{2a}}.$$

61. If, however, we assume that the fringes are produced by the meeting of the direct rays with those reflected at the edge of the opaque screen, and if we suppose further that the reflected rays lose half a wave-length, we have [*section* 20] for the same band

$$x = \sqrt{\frac{2(a+b)b\lambda}{a}} \text{ or } x = 2\sqrt{\frac{(a+b)b\lambda}{2a}}.$$

Accordingly, these two quantities are in the ratio of 2 to 1.873. The second is measurably smaller than the first, differing as they do by nearly a fifteenth; so that by accurate observations on homogeneous light of well-determined wave-length one might distinguish between these two theories by means of experiment.

62. The method which I at first thought best adapted to the determination of the wave-length was to measure the size of the fringes produced by two mirrors slightly inclined to one another, and also the distance between the two images of the luminous point; but the slightest curvature in the mirrors diminishes the accuracy, and so I preferred to use the bands

* [*The general expression for this difference of path, d, has been given above—section* 20.

$$d = \frac{ax^2}{2b(a+b)}.$$

Put $d = \frac{\lambda}{4}$, *and we have the value of x in question.*]

produced by a narrow slit combined with the cylindrical lens of which I have already spoken. We have already found that the distance between any two consecutive dark bands, either to the right or the left of the aperture, is $\frac{b\lambda}{c}$, where λ is the wave-length, c the width of the aperture, and b its distance from the micrometer. The distance between the two dark bands of the first order is just twice this amount. With these data, it is an easy matter to determine λ from measurements on the fringes.

The following table gives the results of five observations of this kind, together with the wave-lengths computed from them. In order to describe all the conditions of the experiment, I include in the table the various values of a, the distance from the luminous point to the screen, even though this quantity is not employed in the calculation. These measures have been made with practically homogeneous red light, obtained by use of the same colored glass which, for the purpose of getting results that are comparable, I have used in all my observations. Each measure recorded in the table is the mean of four observations.

Distance from luminous point to diaphragm a	Distance from diaphragm to micrometer b	Size of aperture	Number of bands $\frac{b\lambda}{c}$ included in each measure	Mean of micrometer measures	Wave-lengths computed from these measures
m.	m.	mm.		mm.	mm.
2.507	1.140	2.00	6	2.185	0.000639
2.010	1.302	4.00	10	2.075	0.000637
2.010	1.302	3.00	8	2.222	0.000640
1.304	2.046	3.00	8	3.466	0.000635
1.304	2.046	2.00	6	3.922	0.000639
				Sum of these results=	0.003190
				Fifth of the sum or mean=	0.000638

The agreement of these results with each other is very satisfactory, differing, as they do, among themselves by less than one per cent. Accordingly I have adopted the value 0.000638 mm., and have employed it in all my comparisons of theory and experiment.

[*Four pages devoted to verifications of the results in this table are here omitted.*]

65. Having thus, by means of simple and well-known methods, verified the wave-length determination made with the single slit and cylindrical lens, I have used this same value to compute the exterior fringes by use of the formula

$$x = n\sqrt{\frac{(a+b)b\lambda}{2a}},$$

substituting for n those values of v which, according to the table, give *maxima* and *minima*.

The table on page 130 summarizes the results of calculation and observation. In my experiments I have measured the positions of the *minima* only, because I considered this a sufficient test of the theory, and because my eye can determine the darkest point of a dark band with greater accuracy than it can set upon the brightest point of a bright band.

[*Only every fifth observation in the table is reproduced.*]

More striking agreement between theory and experiment could scarcely be expected. When these small differences are compared with the quantities measured, and when the great variations in the quantities a and b are noted, one can no longer doubt that the integrals which led to these results accurately describe the law governing the phenomena. The probability in favor of the new theory is still further increased by the fact that the wave-length here employed has been deduced from different and simpler phenomena.

[*Four pages, devoted to a description of some experimental precautions and to a computation of exterior fringes on Young's hypothesis of reflection from the edge of the opaque body, are here omitted.*]

68. We have just seen that both the formation and the position of the exterior fringes can be explained in a satisfactory manner by considering them as produced by the meeting of an infinitely great number of secondary waves which originate on that part of the primary wave which is not intercepted by the opaque screen. From this view it follows that the light which is inflected into the shadow ought not to produce any bright or dark band, but ought to diminish gradually in intensity, provided the screen is sufficiently large to allow no light to go around the other side; and this is true, even though this inflected light, like that which gives rise to the exterior fringes, is the resultant of an infinitude of secondary waves. This will

TABLE COMPARING THE RESULTS OF EXPERIMENT WITH THOSE OF THEORY

EXTERIOR FRINGES IN RED LIGHT OF WAVE-LENGTH 0.000638 MM.

Number of observation	Distance from luminous point to opaque screen a	Distance from opaque body to micrometer b	Order of dark band	Distance from darkest point in each band to edge of geometrical shadow: Observed	Computed	Difference
	m.	m.		mm.	mm.	
			1	2.84	2 83	−1
			2	4 14	4.14	0
1	0.1000	0.7985	3	5.14	5.13	−1
			4	5.96	5.96	0
			5	6.68	6.68	0
			1	1.05	1 05	0
			2	1.54	1 54	0
5	0.510	0.501	3	1.90	1.91	+1
			4	2.21	2 22	+1
			5	2.49	2.49	0
			1	2.59	2 59	0
			2	3.79	3.79	0
10	1.011	2.010	3	4.68	4.69	+1
			4	5.45	5.45	0
			5	6.10	6.11	+1
			1	0.54	0.55	+1
			2	0.80	0.81	+1
15	3.018	0.253	3	1.00	1.00	0
			4	1.16	1.16	0
			5	1.31	1.31	0
			1	3.19	3.22	+3
			2	4.70	4.71	+1
20	3.018	3 995	3	5.83	5.84	+1
			4	6.73	6.78	+5
			5	7.58	7.60	+2
			1	1.13	1.14	+1
			2	1.67	1.67	0
25	6.007	0.999	3	2.06	2.07	+1
			4	2.49	2.49	0
			5	2.69	2.69	0

be easily seen by looking at the table below, which gives the intensity of light in the shadow for rays inflected at various angles. These intensities have been computed by means of the table giving the numerical values of the integrals

$$\int dv \cos qv^2 \text{ and} \int dv \sin qv^2,$$

by taking the sums of the squares of the corresponding numbers and subtracting $\frac{1}{2}$. In spite of the inaccuracy introduced by the method of partial integration employed in the first table, it is seen that the intensity of light diminishes rapidly as v increases, presenting none of the *maxima* and *minima* observed outside of the shadow.

INTENSITIES OF LIGHT DIFFRACTED INTO THE SHADOW UNDER DIFFERENT ANGLES

Values of v	Corresponding intensities	Values of v	Corresponding intensities
0.10	0.4095	2.90	0.0121
0.20	0.3359	3.00	0.0113
0.30	0.2765	3.10	0.0105
0.40	0.2284	3.20	0.0098
0.50	0.1898	3.30	0.0092
0.60	0.1586	3.40	0.0087
0.70	0.1334	3.50	0.0083
0.80	0.1129	3.60	0.0079
0.90	0.0962	3.70	0.0074
1.00	0.0825	3.80	0.0069
1.10	0.0711	3.90	0.0066
1.20	0.0618	4.00	0.0064
1.30	0.0540	4.10	0.0061
1.40	0.0474	4.20	0.0057
1.50	0.0418	4.30	0.0054
1.60	0.0372	4.40	0.0052
1.70	0.0332	4.50	0.0051
1.80	0.0299	4.60	0.0048
1.90	0.0271	4.70	0.0045
2.00	0.0247	4.80	0.0044
2.10	0.0226	4.90	0.0043
2.20	0.0207	5.00	0.0041
2.30	0.0189	5.10	0.0038
2.40	0.0173	5.20	0.0037
2.50	0.0159	5.30	0.0036
2.60	0.0147	5.40	0.0035
2.70	0.0137	5.50	0.0033
2.80	0.0129		

As usual, a and b represent the distances of the screen from the luminous point and from the plane in which the shadow lies, while x is the distance from the edge of the geometrical shadow to the point in this plane under consideration, so that we have

$$x=v\sqrt{\frac{(a+b)b\lambda}{2a}},$$

and therefore

$$\frac{x}{b}=v\sqrt{\frac{(a+b)\lambda}{2ab}}.$$

69. By the aid of these formulæ we can find the value of the distance x or the angle x/b of the inflected ray corresponding to the various values of b; and *vice versa*, if x or the slant x/b be given, we can find v, and thus determine the intensity of the inflected light. One striking inference from this formula, $x=v\sqrt{\frac{(a+b)b\lambda}{2a}}$, is that the values of x are not directly proportional to those of b, but are related to them as the ordinates of a hyperbola are to its abscissas. It thus follows that points of equal intensity along the edge of the geometrical shadow do not lie upon a straight line as we vary b, but upon a hyperbola of appreciable curvature, like the corresponding loci in exterior fringes.

70. I have not yet succeeded in verifying by direct experiment the ratios of intensity in the inflected light as predicted by the theory of interference applied to the principle of Huygens. A measurement of this kind is very difficult [*foot-note omitted*], and I hardly think that one would be able to reach the same accuracy as in the determination of the darkest and brightest points in fringes. The results already obtained for fringes, however, appear to me as verifications—indirect, it must be confessed—of these very ratios of intensity; for whenever the positions of *maxima* and *minima* have been deduced from the general expression for the intensity of light and have been found to coincide accurately with experiment, it becomes more and more probable that this integral correctly represents all the variations of intensity in the inflected light.

71. In the case of exterior fringes one may, as we have seen, use the table of *maxima* and *minima* to compute the positions of the darkest and brightest points in the dark and bright bands for all values of a and b. This, however, is not the case with regard to the interior fringes in the shadow of a narrow body or in the case of a narrow aperture. The limits of the integral vary all the while, and it is therefore impossible to give general results applicable to every case, so that one is obliged to determine the *maxima* and *minima* for each particular case, using

the table, which gives the numerical values of $\int dv \cos qv^2$ and $\int dv \sin qv^2$. I propose to give the results of all the computations of this kind which, up to the present, I have made for the purpose of testing the theory. They are very long, and I have not been able to finish as many as I had desired, but this lack in quantity is, perhaps, compensated by the variety of the cases which I have studied, for in trying the theory on the observations, I have, by preference, selected cases in which the disposition of the fringes is somewhat unusual.

72. And, first, I propose to consider the case of a narrow aperture which presents at once the case of exterior and interior fringes.

Let C be a luminous point, AG a narrow aperture whose edges A and G are straight and parallel; let BD be the central projection of this aperture upon the plane in which the fringes are observed, and P be any point in this plane at which the intensity is to be determined. For this purpose we must integrate

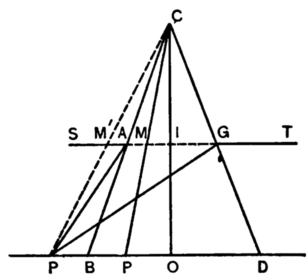


Fig. 20

$$\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right)$$

and

$$\int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right)$$

between the limits A and G, afterwards taking the sum of the squares of these integrals. This will give us the intensity of light at the point P, but we must not forget that the origin from which z is measured lies upon the direct line CP, and that therefore the two limits A and G correspond to $z=$MG and $z=-$AM. The next step is to compute the corresponding values of v from the formula

$$v=z\sqrt{\frac{2(a+b)}{ab\lambda}},$$

or

$$v=x\sqrt{\frac{2a}{(a+b)b\lambda}},$$

where x is the distance from the point P to the edge of the geometrical shadow. From the table of integrals,

$$\int dv \cos qv^2 \text{ and } \int dv \sin qv^2,$$

we then find the values most nearly corresponding to those of v.

Let us call t the difference between the value for which the integral is desired and the number i [*for which it is computed*] in the table. The proper integral can then be found by means of the formulæ of approximation.

$$\int_0^{i+t} dv \cos qv^2 = \int_0^i dv \cos qv^2 + \frac{1}{2iq}(\sin qi\,(i+2t) - \sin qi^2)$$

$$\int_0^{i+t} dv \sin qv^2 = \int_0^i dv \sin qv^2 + \frac{1}{2iq}(-\cos qi\,(i+2t) + \cos qi^2).$$

Having made this computation for the two values of v which represent the edges A and G of the aperture, if the point is inside we add these integrals; if, however, it is on the outside, we subtract them; and, lastly, take the sum of the squares of the two numbers thus found. In like manner, one finds the intensity of the light for any other point whose position is given, and in comparing these various results the positions of *maxima* and *minima* may be found.

[*Half a page concerning the method of interpolation is here omitted.*]

73. In order to apply this method of computation to the observations, I first determined the tabulary value of c, that is to say, the size of the aperture, by means of the formula

$$v = c\sqrt{\frac{2(a+b)}{ab\lambda}}.$$

so that I thus obtained the tabular interval between the limits. By a few easy trials I find between what numbers of the table the *maxima* and *minima* lie; afterwards I determine their position more accurately by the process which I have just described. Having thus obtained the values of v corresponding to *maxima* or *minima*, I subtract them from the half of the tabulary value of c, in order to refer them to the middle of the aperture. And, last of all, the formula

$$x = v\sqrt{\frac{(a+b)b\lambda}{2a}}$$

gives me the distance of these same *maxima* or *minima* from the middle of the projection of aperture, which is the point of reference used in my observations.

COMPARISON OF THEORY AND EXPERIMENT

REGARDING THE POSITIONS OF MAXIMA AND MINIMA IN THE FRINGES PRODUCED BY A NARROW APERTURE

Number of bright or dark bands counted from middle	Approximate value of *v* counted from edge of aperture	Corresponding intensity	Value of *v* corresponding to *maxima* or *minima*	Distance of *maxima* or *minima* from projection of centre of aperture: Computed	Distance of *maxima* or *minima* from projection of centre of aperture: Observed	Difference
FIRST OBSERVATION						
a=2.010 m.; b=0.617 m.; c=0.50 mm.; tabulary value of c=1.288				mm.	mm.	mm.
1. Minimum	+0.812 +0.912 +1.012	0.03495 0.01645 0.03406	+0.913	0.79	0.77	+0.02
2. Minimum	+2.412 +2.512 +2.612	0.00238 0.00235 0.00541	+2.463	1.58	1.58	0.00
THIRD OBSERVATION						
a=2.010 m.; b=0.401 m.; c=1.00 mm.; tabulary value of c=3.062				mm.	mm.	mm.
1. Minimum	−1.262 −1.162 −1.100	2.2575 2.2153 2.2577	−1.181	0.14	0.16	−0.02
2. Minimum	−0.300 −0.262 −0.162	0.7135 0.6925 0.6950	−0.215	0.51	0.48	+0.03
3. Minimum	+0.400 +0.438 +0.500	0.1501 0.1477 0.1604	+0.431	0.77	0.76	+0.01
4. Minimum	+0.938 +1.038 +1.138	0.0799 0.0417 0.0432	+1.084	1.02	1.01	+ .01
5 Minimum	+1.800 +1.738 +1.700	0.0170 0.0128 0.0141	+1.736	1.28	1.28	0
FIFTH OBSERVATION						
a=2.010 m.; b=0.492 m.; c=1.50 mm.; tabulary value of c=4.224				mm.	mm.	mm.
1. Maximum	−1.300 −1.200 −1.100	2.7239 3.0466 2.9780	−1.168	0.42	0.43	−0.01

[*The second, fourth, and sixth observations are omitted.*]

Evidently theory and observations agree in general quite well, although in the second and fourth observations the disagreement is quite marked and rather more than one would expect from the size of the fringes; for in the second observation the individual measures differ at most by 0.04 mm., and the fourth observation, which I have already described, agrees perfectly, as has been seen, with another experiment in which the same fringes appear. This disagreement, therefore, can only be explained by assuming that the theory is wrong or that constant errors have entered the observations through optical illusion.

74. Our theory rests upon a hypothesis which is at once so simple and so inherently probable, and which besides has been so strikingly verified by many varied experiments, that one can scarcely doubt the truth of the fundamental principle. It is quite possible that this anomaly is only apparent, and that the eye does not correctly estimate the position of the *minima* in question. We must remember that they are not very sharp, and that they are always bounded on each side by two bright bands of very different intensities. Now, in order to determine the position of the *minimum,* my eye must include a part of each of these two bands, so that that part of the dark band on the side next the brightest appears to me darker still on account of its environment; thus attracting, as it were, the apparent *minimum* to its side, and, indeed, all the discrepancies lie in this direction. That the eye includes a sufficiently large portion of the fringes for correctly estimating the position of *maxima* and *minima* is evident from the fact that in repeating the fourth observation, using a diaphragm of small aperture in the focus of the micrometer eye-piece, nothing was left but a band which was uniformly dark and in which the minimum was no longer distinguishable. If I have succeeded in getting the correct positions of the minima in the exterior fringes even in regions of poor definition, it is owing to the fact that the bright bands between which these are included differ very slightly in intensity; and if, in the case of the narrow aperture and cylindrical lens, experiment and theory happen to agree in spite of great differences of intensity between two adjoining bright bands, this is because the dark band, especially in the first and second orders, is almost perfectly black. In general, whenever the *maximum* or *minimum* is very sharp, I find ex-

periment and calculation in thorough agreement. In the fifth observation, for instance, I measure the distance from the centre to the *maximum* of the first order because this bright band is very well defined, and I am therefore able to determine its most brilliant point with great precision. The difference between the computed and observed values is indeed only 0.01 mm.

75. But our theory does more than merely give us the positions of *maxima* and *minima*, for it enables us to predict the general appearance of the phenomena, so that, without experimental determination, we can foretell the variations of intensity in the light; thus, for instance, in the fifth observation the part of the shadow corresponding to the middle of the aperture was filled by a large dark band of a tint that was practically uniform up to 0.26 mm. on each side of the centre, after which the intensity of the light increased rapidly so as to form the bright band of the first order which I have just mentioned. Now in computing the intensity of the light within these limits, we find that in fact its intensity varies scarcely at all, but that in passing from these limits to the bright band it increases very rapidly. In the following table are given the results of computation for different points of the dark band and the two bright bands which include it. The position of each point is denoted by the corresponding value of v, measured always from one of the edges of the aperture.

	Number of observation	Value of v	Intensity
	1	1.100	2.9780
	2	1.200	3 0466
	3	1.300	2.7239
	4	1.400	2.2843
Limit of the colored region as determined by observation.	5	1.524	1.9671
	6	1.824	1.9100
	7	2.112	1.9802

The distribution of intensities on the other side of the centre is the same.

If the distances of these various points from a common origin be plotted as abscissas and the corresponding intensities as ordinates, we shall obtain the curve MCM′, which gives us in fact a picture of the phenomenon just as one finds it in the

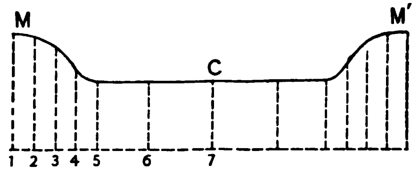


Fig. 21

experiment. I should like to have made similar drawings for all the other observations in order to facilitate the comparison of theory with experiment, but the length of the computation and the time at my disposal did not permit.

[*Five pages, in which the case of a narrow opaque obstacle is discussed, are here omitted.*]

79. I have now applied the principle of Huygens to the three general classes of phenomena in which diffraction occurs, namely, first, to the fringes produced by a screen whose edges are straight and infinitely long, and which is so large that the light passes practically only one edge of the screen; secondly, to the fringes produced by a system of two similar screens brought very near together; thirdly, to those fringes which accompany the shadow of a very narrow screen.*

Comparing observations with the predictions of the theory, I have shown that it suffices to explain the most diverse phenomena, and that the general expression for the intensity of light derived from it gives us a faithful picture of the phenomena, even when they are most bizarre and apparently irregular.

Besides the three general classes, one might devise a large number of others by combining these among themselves. The theory would doubtless apply here with the same success and the same ease. The computation would be more tedious in proportion as the variety of limits assigned to the integrals became greater and greater; the experiments would also demand more complicated apparatus.

80. In the first section of this memoir I have described a phenomenon which results from a combination of two of the principal cases of diffraction, namely, the fringes produced by

* I do not here include those fringes which are produced by the biprism, or two mirrors slightly inclined to each other, for, properly speaking, these are not diffraction effects, since they are not produced by rays which are diffracted or inflected, but by two pencils which are regularly re flected or refracted.

light in passing through two apertures, each very narrow and each near to the other. Having prepared a sheet of copper in the form drawn in Fig. 15, I noted that when the large fringes produced by each of the slits CEC′E′ and DFD′F′, expanding as I moved away from the screen, had filled the shadow of CDFE so that it contained only the bright band of the first order, the interference bands resulting from the two pencils of light became much sharper and brighter than the interior fringes of the part ABCD. The lower part, CEDF, which was at first brighter than the other, became darker the farther I went away from the screen, but its fringes continued to show colors which in white light were purer and bands which in homogenous light were sharper. With the simple apparatus which I employed one could not obtain exact measures, and I have not therefore carried out the computations for this experiment; accordingly I limit myself to the explanation of these phenomena by means of some general considerations.

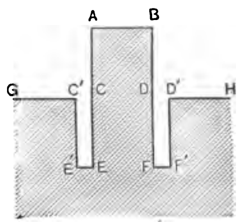


Fig. 15

Let L be the luminous point, and IK the horizontal projection of the part AEBF of the screen represented in Fig 15. P is any point in the interior of the shadow lying upon the straight line LO. From the point L as centre, and with a radius equal to LI, describe the arc IMM′, representing the incident wave. Now with a point P as centre, and with a radius equal to IP, describe the arc I*mm*′. The various distances between these two arcs give us the differences of path traversed by the secondary waves meeting at P. We shall first consider the upper part of the screen—that is, the case where the wave IMM′ is not intercepted on the other side of the point I. Let us now imagine this wave divided into a large number of small arcs, IM, MM′, etc., in such a manner that the straight lines drawn to P from any two consecutive points of division differ by half a wave-length; and, for sake of simplicity, let us suppose that the

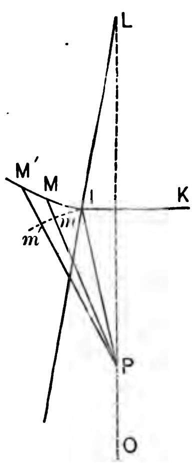


Fig. 22

point P lies well within the edge of the shadow, or, what is the same thing, let us imagine the ray IP sufficiently inclined to the incident ray to make these arcs practically equal. Then each of these arcs, excepting the one at the end IM, will lie between two others, which will combine to annul its effect at the point P. In the case of the arc IM, which lies at the extreme edge of the wave, we have, however, an exception ; for this arc loses only one-half its intensity by interference with the vibrations of the neighboring arc, MM′. If, therefore, we intercept this arc [MM′] and all the rest of the incident wave, the light which is received by the point P will actually be increased ;* this is precisely the effect which, at a certain distance, is produced by the part of the screen G′C′E″ (Fig. 15). But in proportion as the point P (Fig. 22) recedes from the opaque screen, the arc *Imm′* approaches the wave IMM′; and in the case where the luminous point L is at an infinite distance, these two approach indefinitely near to each other. The divisions M, M′, etc., being determined by the separation of these two arcs, keep spreading apart from the point I in proportion as the arcs approach each other. It follows, therefore, that the part MI of the incident wave will grow larger and larger, and the rays from this part passing the edge C (Fig. 15) retain at least half their intensity in the region behind the upper part of the screen. But in the lower part of the screen the aperture *CEC′E′* does not increase in size, so that if the luminous point is far enough away the effective arc IM (Fig. 22) will finally become so large compared with this aperture that [*most of the rays from* MI *are intercepted by* GC′E′, *and hence*] the point will receive less light in the lower part of the shadow than in the upper.

Let us now pass to the consideration of fringes produced by the meeting of rays coming from both edges of the screen, AEBF (Fig. 15). Behind the upper part, ABCD, the inflected light diminishes rapidly in intensity as one recedes from the edge of the geometrical shadow, and therefore all the fringes except those which are very near the middle are produced by two rays of very unequal intensities; consequently the dark

* The light at P would be increased still more if the screen were perforated in such a way as to permit all the arcs of even order to pass through and at the same time intercept all the arcs of odd order.

bands are not very sharp when one uses homogeneous light, and the colors are mixed with gray when one uses white light. Behind the lower part, CEDF, the two pencils of light coming from the slits CEC′E′ and DFD′F′ have a practically uniform intensity throughout a considerable portion of the bright band from each of these apertures; and if these apertures are so narrow compared with the distance between them that the region of uniform intensity in the inflected light includes all the fringes produced by the two pencils, then in those points where the vibrations are in complete discordance the light-waves will almost completely destroy one another; accordingly the dark bands will be very much sharper than in the upper part of the shadow when homogeneous light is employed, and the colors will be very much purer when white light is used. When one looks at these points close up to the screen before the larger fringes which are produced by each slit have spread out into the shadow AEBF, the phenomenon becomes very complicated and changes rapidly with the distance of the magnifying-glass, especially when the distance between the two slits is not very great when compared with their size. It would be interesting to determine by computation the positions of the *maxima* and *minima* of the bright and dark bands, and to compare these results with those of observation. I have no doubt that the theory would thus acquire fresh confirmation.

81. Hitherto we have considered all waves as coming from a single centre, but, in actual experiment, luminous points are always made up of a very large number of centres of vibration, and it is to each one of these by itself that the preceding discussion applies. So long as these are not very widely separated from each other, the fringes which they produce practically coincide, but the dark bands from one overlap the bright bands of the other in proportion as we increase the dimensions of the luminous point, until finally they completely annul each other. In the case of the exterior fringes this effect is more and more appreciable as one gets farther and farther away from the screen, because it increases directly as the distance, while the size of the bright and dark bands increases less rapidly. And this is why a luminous source sufficiently small to produce fringes which, in the near neighborhood of an opaque body, are very sharp will, at a considerable distance from this body, give only ill-defined fringes.

82. It is not necessary that the interposed body should be opaque in order to produce the phenomena of diffraction at its edges; all that is required is that a part of the wave should be retarded with respect to its neighboring parts, but this is exactly what a transparent body does when its refractive index differs appreciably from that of the medium surrounding it; it thus gives rise to fringes which border both the inside and the outside of their shadow. They are exactly like the exterior fringes of opaque bodies when the difference of path between the rays which have traversed the transparent screen and the outside rays contains a considerable number of wave-lengths, because their mutual influence [*interference*] is no longer appreciable and we have simply the addition of two uniform illuminations. But this is not the case when the transparent screen is very even or when its refractive index differs very slightly from that of the surrounding medium, for now the fringes are altered in a very marked way by the mutual influence of those rays which traverse the transparent plate and those which pass its edge. It is from similar reasons that the striæ in layers of mica resulting from slight differences of thicknesses give rise, in white light, to colored fringes in the very peculiar manner described by M. Arago.

83. As to fringes of the kind which we have called interior, they are not to be obtained with a narrow transparent body, because the direct light which traverses it is so much brighter than the inflected rays as to mask the effects of interference; and, besides, the bright and dark bands which this transparent body tends to produce, when considered as a narrow aperture, do not coincide with those which it tends to produce when considered as a small obstacle.

84. The phenomena of diffraction, once explained for the case of homogeneous light, are easily predicted for the case of white light. These fringes come from the superposition of all the bright and dark bands of the various sizes produced by the different kinds of waves which go to make up white light, so that when we have once computed the intensity of each of the principal kinds of rays at the point under consideration, using the proper wave-length, according to the theory which I have just explained, we can find the resultant tint by substituting these values in Newton's empirical formula for determining the result obtained by mixing any set of colored rays.

85. Polished surfaces illuminated by a point-source present a set of diffraction phenomena exactly like those which we observe in direct light. The field of light reflected by a mirror is bordered with fringes similar to those which surround the shadows of bodies. If the surfaces be very narrow or so blackened that only a single bright line remains, or indeed if one inclines the mirror in such a way as to diminish greatly the size of the field [*foot-note here omitted*], the phenomenon of a pencil of light dilated by passing through a very narrow aperture will be reproduced. If a mirror be blackened throughout its entire extent, with the exception of two bright lines, it gives rise to a set of fringes identical with those produced by two parallel slits in an opaque screen. If, instead of blackening the large part of the reflecting surface, one, on the contrary, merely traces a single fine black line, it will produce fringes similar to those observed in the shadow of a narrow screen. In short, the phenomena are absolutely the same as if the mirror were transparent and the rays came from the image of the luminous point. The explanation is very simple; for we know that the image (which lies upon the perpendicular drawn from the luminous point to the mirror, and which is situated at a distance from the surface of the mirror equal to the distance of the luminous point from the mirror), has this remarkable property, namely, that its distance from any point on the surface of the mirror is equal to the distance of the same point from the luminous centre. When, therefore, we consider the rays as originating in the image of a luminous point, we do not alter the difference of path traversed by the elementary waves which produce the fringes, and consequently there is no change in the size, or in the relative intensities, of the bright and dark bands.

I may here remark that the position [*phase*] of the resultant of the secondary waves at any point, depending as it does merely upon differences of path, ought, in the case of reflection, to be the same as if the rays were emitted by the image just mentioned. Consequently, in the case of a polished surface of large area, all the partial resultants will be situated at the same distance from this point, thus making it the centre of the reflected wave.

86. It is by means of these secondary waves that Huygens has explained in such a simple manner the laws of reflection

and refraction, showing that they are phenomena of the same kind as the propagation of light in a homogeneous medium; but his explanation leaves much to be desired. He has not proved that there will be only one system of waves resulting from this multitude of systems of secondary waves, for he has not used the principle of interference. He assumes that the light is appreciable only in those points where the secondary waves coincide [*in phase*] exactly; while the complete absence of any luminous disturbance can occur only when the secondary disturbances are in [*direct*] opposition. It was this, doubtless, that led him to think that light was not inflected to any appreciable extent into shadows, and which prevented him from discovering the phenomena of diffraction, the laws of which his theory could have given him without recourse to experiment.

This theory, when combined with the principle of interference, gives us not only the path of the ray in the particular case where reflection occurs at a polished surface of indefinite extent, but also in those cases where the surface is very narrow or even discontinuous; it shows us how diminution in size of the surface produces the dilation of the reflected ray, and how a system of very narrow mirrors placed side by side and very close together can produce colored images, owing to the mutual influence of pencils of light thus dilated. This is the phenomenon of ruled surfaces. With the same case it explains the images and colored rings produced by a thin fabric or even an irregular combination of very fine threads or small particles, provided they are almost equal in size, when placed between the eye of the observer and the luminous point.

I think it hardly necessary to emphasize these phenomena, since they are merely combinations of those described above, and since I have attempted to give, for all of them, a general theory.

ON THE ACTION OF RAYS OF POLARIZED LIGHT UPON EACH OTHER*

BY

ARAGO AND FRESNEL

1. Before describing the experiments which form the subject of this memoir it will perhaps be well to recall the exquisite results obtained by Dr. Thomas Young, who, with rare sagacity and characteristic skill, has already studied the effects which rays of light exert upon each other.

First. Two rays of homogeneous light coming *from the same source* and reaching a certain point in space by paths which are *different and slightly unequal* in length, either strengthen one another or annul one another, and produce upon the receiving screen a bright or a dark point according as the difference of path has one value or another.

Second. Two rays always intensify each other at any point for which their paths are equal; if their intensities are added for another point where the difference of path is equal to a quantity d, their intensities will be added also for all differences of path included in the series $2d$, $3d$, etc. The intermediate values $0+\frac{1}{2}d$, $d+\frac{1}{2}d$, $2d+\frac{1}{2}d$, etc., represent the points in which the rays annul each other.

Third. The quantity d does not have the same value for all homogeneous rays. In air its value for the extreme red rays of the spectrum is $\frac{66}{100000}$ mm., while for violet rays it is only $\frac{42}{100000}$ mm. For other colors the corresponding values are intermediate between these which we have just given.

The periodicity of color which is seen in Newton's rings, in

* [*Annales de Chimie et de Physique*, t. x., p. 288 (1819).]

halos, etc., seems to depend upon the influence exerted upon one another by rays whose paths at first diverge, and later are so inclined as to again meet; but in order to bring these various phenomena into harmony with the laws just stated, we are forced to *admit* that difference of path alone is not sufficient to determine the mutual action of two rays at their point of meeting except when they are both travelling in the same medium; and it must be recognized also that differences of refractive index, or thickness in the transparent bodies traversed by the respective rays, produce the same effect as difference of path. In this journal, vol. i., p. 199, there is described a *direct* experiment due to M. Arago, which shows the same thing, and proves also that a transparent body *diminishes* the speed of light traversing it in the ratio of the sine of the angle of incidence to the sine of the angle of refraction; so that in all the *phenomena of interference* * two different media produce similar effects when their thicknesses are in inverse ratio to their refractive indices. These considerations at once suggest a new method for measuring slight differences of refrangibility.

2. While we were trying to determine what accuracy was attainable by this method, one of us (M. Arago) thought that it would be interesting to find out whether the actions which ordinary rays exert one upon another were in any way modified when two previously polarized pencils of light were made to interfere. We know that if a narrow body be illuminated by light coming from a point-source, its shadow is bordered on the outside by a series of fringes produced by the interference of the direct light with the rays inflected near the opaque body. It is known also that a part of this same light, passing into the geometrical shadow from the two opposite sides of the body, there gives rise to fringes of the same kind.

Now the fact was easily recognized that these two systems of fringes are absolutely the same, whether the incident light has received no modification whatever, or whether it has been polarized previous to incidence. Rays which are *polarized, in one plane,* therefore mutually affect one another in the same manner as rays of ordinary light.

* This is the name which Mr. Young has given to the phenomena produced by the meeting of two or more rays of light.

3. It was still to be determined whether two rays *originally polarized* at right angles would not produce phenomena of the same kind when they met inside the geometrical shadow of an opaque body. For this purpose we placed in front of the point-source* sometimes a rhomb of calc-spar, sometimes an achromatic prism of rock crystal, and thus obtained two luminous points. In each case we had a divergent pencil, and these two pencils were polarized at right angles. Behind the two radiant points and midway of the space between them was placed a cylinder of metal. In this manner a part of the polarized light from the first pencil reached the interior of the shadow *via* the right-hand side of the cylinder; while a part of the light from the second pencil, polarized in a plane at right angles to the first, entered the shadow from the left-hand side of the cylinder. Some of these rays met along the line joining the centre of the cylinder and the middle point of the straight line drawn from one luminous point to the other. Here these rays had traversed equal paths, and one might expect them to produce fringes. On the contrary, not the slightest trace of fringes could be seen, even with a magnifying-glass. In fact, the rays here cross without either affecting the other. The only fringes which make their appearance in this experiment arise from the interference of rays which come from only one of the radiant points and enter the shadow from each side of the cylinder. Those which we tried to produce by the interference of rays polarized at right angles to each other would have occupied a position intermediate between those just mentioned.

Since the images which we employed were not very widely separated, the thicknesses of crystal traversed by the ordinary and the extraordinary rays must have been very nearly equal. Nevertheless, similar experiments have already shown us, only too frequently, how sensitive the phenomena of interference are to the slightest difference of speed in the rays, to the length of path, and to the refractive index of the medium. No argument was needed, therefore, to convince us of the necessity of repeating these experiments under conditions which would eliminate these various sources of inaccuracy. This has been attempted by each of us.

* For all the experiments described in this paper our source of light was the focus of a small magnifying-glass.

4. M. Fresnel at once devised two distinctly different methods. The principle of interference shows us that pencils of light from two luminous points, originally from a single point-source, produce bright and dark bands at points of intersection even though no opaque body be interposed. (See *Annales de Chimie et de Physique*, t. i., p. 332.)

To solve the problem it is then only necessary to determine whether, when *two* images are produced by placing a rhomb of calc-spar in front of a luminous point, they will behave in this same way; but since, from the theory of double refraction, we know that the extraordinary ray traverses carbonate of lime more rapidly than the ordinary ray, it becomes necessary to compensate this extra speed before the two rays are allowed to intersect. In order to accomplish this a method was employed which has been described by M. Arago in this journal, vol i., p. 199. M. Fresnel placed in the path of the extraordinary pencil *alone* a plate of glass whose thickness had been determined by computation in such a way that, under perpendicular incidence, this pencil lost nearly all the ground which, in the crystal, it had gained over the ordinary ray. By slightly inclining the plate the compensation could be made exact. In spite of these precautions, the two rays, polarized at right angles, gave not the slightest trace of interference bands.

In another experiment, M. Fresnel compensated for the difference of speed in the two rays by allowing them each to fall upon a small unsilvered mirror whose thickness had been so *computed* that the extraordinary ray, when reflected at the second face, lost by twice traversing the glass more than it had gained in traversing the crystal; a gradual inclination of the plate brought about complete compensation.

Under no angle of incidence, however, would the ordinary rays, reflected at the first surface, interfere with the rays reflected from the second surface to produce bands.

5. In order to avoid the theoretical consideration introduced into the preceding experiment, and to maintain the original intensity of the light, M. Fresnel adopted the following method: A rhombohedron of calc-spar was sawed through the middle, and the two parts were placed one in front of the other with their principal sections at right angles. In this position, the ordinary ray from the first crystal was refracted as an extraordinary ray in the second; while, conversely, the extraor-

dinary ray in the first crystal suffered ordinary refraction in the second. On viewing a luminous point through this combination, one sees only a double image. Each pencil has experienced in succession the two kinds of refraction. The sum of the paths of each pencil through the two crystals ought, therefore, to be equal, since by hypothesis the crystals have the same thickness; so that everything is compensated, both as regards speed and length of path. Nevertheless, two systems of rays polarized at right angles never gave rise to any interference fringes. Lest the two parts of the rhombohedron did not have quite the same thickness, we took pains in each test to vary slightly and slowly the angle of incidence at the face of the second crystal.

6. The method devised by M. Arago for solving this same problem was independent of double refraction. It has been known for a long time that if one cuts two very narrow slits close together in a thin screen and illuminates them by a single luminous point, there will be produced behind the screen a series of bright bands resulting from the meeting of the rays passing through the right-hand slit with those passing through the left. In order to polarize at right angles the rays passing through these two apertures, M. Arago at first thought of using a thin piece of agate, sawed through the middle and placed one piece in front of each slit, in such a way that the edges formerly meeting along the line of the cut are now at right angles to each other. This arrangement ought certainly to produce the effect expected. But not having at hand a suitable piece of agate, M. Arago proposed to supply its place by two piles of plates, of proper thickness, built up from sheets of mica.

For this purpose we selected fifteen plates as clear as possible and superposed them. This pile was next cut in two by use of a sharp tool. So that now we had two piles of plates of *almost exactly* the same thickness, at least in those parts bordering on the line of bisection; and this would be true even if the component plates had been perceptibly wedge-shaped. The light transmitted by these plates was almost completely polarized when the angle of incidence was about thirty degrees. And it was exactly at this angle that the plates were inclined when they were placed in front of the slits in the copper screen.

When the two planes of incidence were parallel, *i. e.*, when the plates were inclined in the same direction, up and down, for instance, one could very distinctly see the interference bands produced by the two polarized pencils. In fact, they behave exactly as two rays of ordinary light. But if one of the piles be rotated about the incident ray until the two planes of incidence are at right angles to each other, the first pile, say, remaining inclined up and down while the second is inclined from right to left, then the two emergent pencils will be polarized at right angles to each other and will not, on meeting, produce any interference bands.

The pains we took to make these two piles of equal thickness would indicate that we also took care in placing them before the slits to have the light traverse those parts which were originally contiguous. But all difficulties of this kind are really solved by the fact that the two rays when polarized in the same plane interfere like ordinary light. Moreover, we could not produce interference by slowly and gradually changing the inclination of one of the plates so long as the planes of incidence were at right angles.

7. The same day that we tried the combination of these two piles we also tried an experiment suggested by M. Fresnel, an experiment which, it must be confessed, is less direct than the preceding, but which is also more easily performed and which demonstrates equally well the impossibility of producing fringes by bringing together rays polarized at right angles to each other.

In front of a sheet of copper in which are cut two slits we placed, for instance, a thin plate of selenite. Since this is a doubly refracting crystal, there will be two pencils of light polarized at right angles passing through each slit. Now if rays polarized in one plane can affect rays polarized in a plane at right angles, we should expect with this arrangement to see three distinct systems of fringes. The *ordinary* rays from the right-hand slit would combine with the *ordinary* rays from the left-hand slit to form a first system symmetrical with respect to the line bisecting the space between the two slits. The bands formed by the two *extraordinary* pencils would fall in the same position as the preceding, increasing their intensity, but remaining indistinguishable from them. As to those which would result from the action of the *ordinary* rays from the right upon the *extraordinary* from the left, and conversely, it is clear that they would form a system to the right and to the left of the central band. The distance of either of these systems from the centre would increase with the thickness of the plate, for, as we have seen, difference of speed is quite as effective as difference of path in shifting the position of fringes. Now, since the fringes in the centre are the only ones visible, even though the plate of selenite be so thin as not to shift the other two systems very much, we must conclude that rays of light polarized at right angles do not affect one another.

8. In order to verify this conclusion, suppose that we cut the selenite plate in two, and that we place one half in front of the first slit and the other half in front of the other slit; and instead of placing their axes parallel as in the case of a single plate, let us put them at right angles to each other. In this way the *ordinary* ray coming through the right-hand slit will be polarized in the same plane as the *extraordinary* ray from the left-hand slit, and *vice versa.* These rays will then form fringes; but their speeds in the crystal will not be equal, and they will not lie symmetrically about the middle of the space between the two slits. Central fringes will be produced only by ordinary or extraordinary rays from the one slit meeting rays from the other slit which are polarized in the same plane. But when the two parts of crystal are arranged as we have here supposed them, those rays which are polarized at right angles to each other ought not to affect one another. One would, therefore, see simply the first two systems of fringes separated by an interval of white or of some uniform shade.

[*An unimportant foot-note is here omitted.*]

If, without changing the experiment in any other respect, we simply set the two plates of selenite so that their axes make an angle of 45° instead of 90°, we should at once see three systems of fringes; for now, since their planes of polarization are no longer at right angles, each pencil from the right will interfere with the two pencils from the left, and *vice versa.* It should be observed also that the middle system is the most intense, resulting, as it does, from the exact superposition of interference bands of polarized pencils of the same kind.

9. Let us return to the combination of the two piles and imagine that the planes of incidence are mutually perpendicular, so that the two pencils are polarized at right angles to each

other. Between the copper screen and the eye place a doubly refracting crystal in such a way that its principal section makes an angle of 45° with the planes of incidence. In accordance with the well-known laws of double refraction, the rays which are transmitted by the piles will afterwards, in passing through the crystal, each be divided into two others. These two will be of equal intensity, will be polarized in planes which are mutually perpendicular, and one of these planes will coincide with the principal section of the crystal. One might therefore expect to see, in this experiment, one series of fringes due to the meeting of the ordinary pencil from the right with the ordinary pencil from the left, and a second series similar to the preceding, but arising from the interference of the two extraordinary pencils. Such, however, is not the case; for these four pencils meet and produce only a uniform illumination, showing not the slightest interference.*

This experiment shows that two rays originally polarized at right angles to each other may subsequently be brought into the same plane of polarization without again acquiring the power of interference.

10. In order to produce interference between two rays polarized at right angles and afterwards reduced to the same plane it is necessary that they should originally have been polarized in one and the same plane. This is shown by the following experiment, which was devised by M. Fresnel.

A plate of selenite, backed with a sheet of copper in which two apertures have been made, is illuminated by a pencil of polarized light coming from a point-source and striking the selenite plate at perpendicular incidence. The axis of the plate makes an angle of 45° with the original plane of polarization. As in all similar experiments the shadow of the copper screen is observed with a magnifying-glass; but in this case

* If the plate interposed between the copper screen and the eye were so thin as to only slightly separate the two images, one might explain the absence of interference as follows: *viz.*, suppose the two systems of bands are superposed in such a fashion that the bright bands of one system coincide with the dark bands of the other system, and *vice versa*. But the insufficiency of this explanation is shown by placing a rhombohedron of Iceland spar between the eye and the preceding crystal. In certain positions this Iceland spar separates the two systems of bands, because they are polarized at right angles. But even under these circumstances one sees no trace of bands.

a rhombohedron of Iceland spar, in which [*the separation of images due to*] double refraction is *perceptible,* is placed in front of the focus.

The principal section of the Iceland spar makes an angle of 45° with that of the plate of selenite. Accordingly we find in each image three systems of fringes, one falling exactly in the middle of the shadow, the others being situated on the right and left respectively.

Let us now consider one of these two images, say the ordinary, and see what gives rise to these three systems of bands.

The pencils which pass through the two slits are polarized in the same plane, but on emergence from the plate of selenite they are divided into two pencils polarized at right angles. Since double refraction in this plate is inappreciable, the ordinary and extraordinary pencils each follow practically the same route, though with different speeds.

Each of these double pencils, say the one from the right-hand slit, will be divided, in passing through the Iceland spar, into four pencils, two ordinary and two extraordinary; but, as a matter of fact, one will see only two, since components in the same plane will coincide. It is also evident, from the well-known laws of double refraction and from the relative positions of the selenite and the Iceland spar, that at emergence from this latter crystal the ordinary pencil will be composed partly of the ray which was ordinary in the selenite and partly of the ray which was extraordinary; while the other two components of these same rays go to form the extraordinary image which we are not now considering. The pencil which emerges from the left-hand slit behaves in the same way. We see, in fact, that the ordinary pencil coming either from the right or left hand slit will, after traversing the two crystals in this new instrument, be composed partly of light which has followed the ordinary path in each crystal and partly of light which started out as an extraordinary ray.

Rays coming from the two slits and following the ordinary path through each of the two crystals will have traversed routes of the same length and with the same speed. On meeting, they ought, therefore, to give rise to central bands. The same is true of rays which have pursued the extraordinary path both in the selenite and in the Iceland spar. The bands in the middle of the shadow result, therefore, from the superposition of these two different systems.

Now as to that portion of light from the right-hand slit which has traversed the selenite as an extraordinary ray, for instance, but passed the Iceland spar as an ordinary ray, it is evident that it will have traversed a path which in length is equal to that of the left-hand pencil which made the whole trip as an ordinary ray. But since in the selenite the speeds are different, those points where they meet to form fringes will not lie symmetrically between the two slits, but will be shifted to the right, *i. e.*, to the side opposite the ray which for a while travelled as an extraordinary ray, but now travels more slowly. Finally, as a last combination, we have interference between that component of the right-hand pencil which traversed both crystals as an ordinary ray and that component of the left-hand pencil which in the selenite was an extraordinary ray and in the Iceland spar an ordinary ray. This interference gives rise to a system of bands situated on the left of the centre.

We have now explained the paths of the rays which meet to form the three systems of fringes in the experiment under discussion. And it may be remarked that the right and left systems were produced by the interference of rays which were previously polarized at right angles in the selenite and afterwards reduced to the same plane in the Iceland spar. Two rays polarized at right angles and later reduced to the same plane of polarization can, then, meet and produce interference bands; but for this purpose it is an essential condition that the rays should ORIGINALLY *have been polarized in the same plane.*

So far we have not considered the interaction of the two pencils which suffered extraordinary refraction in the Iceland spar. These pencils also furnish three systems of bands, but they are separated from the others. If we allow all the conditions of the experiment to remain the same, except that we substitute for the Iceland spar a plate of selenite or quartz which does not give two distinct images, the six systems, instead of being reduced to three by superposition, will result in one central system. This remarkable fact shows, *first*, that the fringes resulting from the interference of the ordinary rays are complementary to those produced by the interference of the extraordinary rays; and, *secondly*, that these two systems are so located that a bright band in the one system corresponds to a dark band in the other system. Were these two conditions not satisfied, one would not find uniform and continuous illumination on each side of the central fringes. We meet here, then, the same difference of half a wave-length that is found in the phenomena of colored rings.

From the experiments just described we may, therefore, infer the following facts:

(1.) Two rays of light *polarized at right angles* do not produce any effect upon each other under the same circumstances in which two rays of ordinary light produce destructive interference.

(2.) Rays of light polarized in the same plane interfere like rays of ordinary light; so that in these two kinds of light the phenomena of interference are absolutely identical.

(3.) Two rays which were *originally polarized at right angles* may be brought to the same plane of polarization *without thereby acquiring the ability to interfere.*

(4.) Two rays of light polarized at right angles and afterwards brought into the same plane of polarization interfere like ordinary light provided they were originally polarized in the same plane.

(5.) In the phenomena of interference produced by rays which have experienced double refraction the position of the interference bands is determined not only by difference of path and difference of speed, but in some cases, as above indicated, it is necessary to take into account also a difference of one-half a wave-length.

All these laws are, as we have seen, based directly upon experimental evidence. In starting from the phenomena of crystalline plates, they can be derived more simply; but then we have to assume that the colors of the plates when illuminated by polarized light are produced by the interference of several systems of waves. The evidence which we have just presented has the advantage of establishing the same laws quite independently of hypothesis.

MEMOIRS ON

BIOGRAPHICAL SKETCH

AUGUSTIN JEAN FRESNEL was born in Normandy in 1788, and died near Paris in 1827.

As a child he was quite the reverse of precocious; but at the age of sixteen he was ready to enter the *École Polytechnique* at Paris, where he received sound mathematical training and attracted to himself the attention of Legendre. His education was completed at the *École des Ponts et Chausées* where he received an engineer's training. Several years were next spent in professional work in various parts of France.

In 1816, through the influence of Arago, he received an appointment in Paris, where he remained during the rest of his life. When we recall that his first studies in optics date from 1814, his accomplishments during the eleven years of his Paris residence must ever fill us with wonder. New ideas were not only rapidly acquired, but were also rapidly perfected. They were at once submitted to the test of experiment and as quickly received elegant mathematical description.

The wave-theory of light had lacked neither merit nor able support; Grimaldi, Hooke, Huygens, and Young had been its advocates; but it was only in the hands of Fresnel that the problem and its solution received such clear and simple statement as to command acceptance. The work of Fresnel lies exclusively in the domain of optics, each of his investigations falling into one of two distinct groups, *viz.*, the kinematics of light and the dynamics of light.

His earlier papers deal with questions of diffraction, interference, and polarization, in which the chief factors of the discussion are displacements, velocities, and squares of velocities —the quantities of kinematics.

His later papers, however, refer more to the medium through which luminous energy is transferred; they deal with the forces of elasticity here brought into play, and seek to determine the speed of light as a function of the mechanical properties of the matter through which the light travels; they deal, in short, with the dynamics of light.

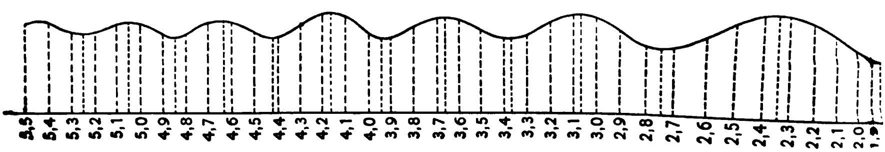
But the particular achievements with which the name of Fresnel must always be associated are

(1.) The introduction of the idea of transverse vibrations.

(2.) The combination of the principle of Huygens with that of interference.

An excellent and appreciative sketch of Fresnel will be found in Arago's *Notices Biographiques*, vol. i. It is here that he paraphrases Newton's remark concerning Cotes by saying "*que nous savons quelque chose quoique Fresnel ait peu vécu.*"

Between the years 1866 and 1870 the French government published the works of Fresnel in three worthy quarto volumes, ably edited by Senarmont, Verdet, and the author's brother, Léonor Fresnel.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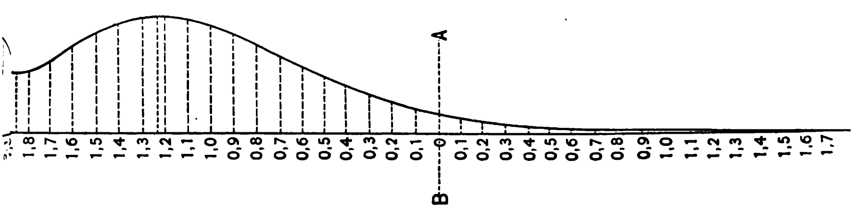
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